### Problem 1
(a) Find the eigenvalues by multiplying each eigenvector by $A^T A$: 64, 4, and 0.
(b) The singular values are the square roots of the nonzero eigenvalues of $A^T A$: 8 and 2.
(c) The SVD is

$$A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

### Problem 2
(a) False. The $2 \times 2$ identity matrix is symmetric, but it has plenty of non-perpendicular eigenvectors, e.g., $(1, 0)$ and $(1, 1)$.
(b) True, because $A$ is diagonalizable using an orthogonal matrix: $A = Q \Lambda Q^T$. Such a matrix is symmetric: $(Q \Lambda Q^T)^T = Q \Lambda Q^T$.
(c) False. Same example as in part (a).

### Problem 3
There is no such value of $d > 0$. To have positive eigenvalues means that $A$ is positive definite. The upper left determinants are $1$, $d - 4$, and $12 - 4d$. These are never all positive.

### Problem 4
(a) $\lambda = 1$ is repeated. The number of independent $\lambda = 1$ eigenvectors is given by the dimension of $N(A - I)$, which is two. So $A$ has two independent $\lambda = 1$ eigenvectors, so $\lambda = 1$ must be repeated.
(b) $A$ has three independent eigenvectors, so it is diagonalizable, i.e., similar to

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

(c) The matrix

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

has the same eigenvalues and number of independent eigenvalues as $A$, so is similar to $A$.
(d) The matrix

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

has the same eigenvalues as $A$ but is missing an eigenvector: the rank of $C - I$ is two, so $C$ has only one independent $\lambda = 1$ eigenvector.