

## 18.06 - Spring 2004 - Problem Set 2

February 11, 2004

This problem set on lectures 4 – 6 is due Wednesday (February 18th), at 4 PM, at 2-106. Make sure to include your **name and recitation number** in your homework! The numbers of the sections and exercises refer to “Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.”.

Lecture 4:

- **Read:** book sections 2.5 and 2.6.
- **Work:** book section 2.5 (exercises 6, 8 and 23) and 2.6 (6, 10, 13, and 20).

Lecture 5:

- **Read:** book section 2.7.
- **Work:** book section 2.7 (exercises 4, 12, 13, 17, 36 and 40).

Lecture 6:

- **Read:** book section 3.1.
- **Work:** book section 3.1 (exercises 5, 10, 18, 23 and 24).

### Challenge Problem

This is about the important  $n \times n$  tridiagonal matrices  $K_n$  and  $T_n$  with  $-1, 2 - 1$  on all the middle rows. The  $(1, 1)$  entry of  $T_n$  is 1 when the  $(1, 1)$  entry of  $K_n$  is 2:

$$K_4 = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}; \quad T_4 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix};$$

- Write MATLAB commands that will create  $K_5$  and  $T_5$  (the *toeplitz* command is useful, or *diag(v,k)* puts a vector  $v$  on diagonal  $k$ ).
- Factor  $K_4$  and  $T_4$  in the form  $LDL^T = (\text{lower triangular})(\text{diagonal pivots})(\text{lower triangular})$ . You can do it by hand, or by using *lu* or *chol* commands.

- c) What is  $L^{-1}$  for  $K_4$  and  $T_4$ ?
- d) What are the inverse matrices  $K_4^{-1}$  and  $T_4^{-1}$ ?
- e) Find a formula (You don't need to prove it, if you don't want) for the  $(i, j)$  entry of  $T_n^{-1}$  (the easy one) and  $K_n^{-1}$  (a little harder, notice each row of  $K_4^{-1}$  and  $K_5^{-1}$  on and above the diagonal).

Please staple your solution as first page of your homework.