18.06 - Spring 2004 - Problem Set 2

February 11, 2004

This problem set on lectures 4 – 6 is due Wednesday (February 18th), at 4 PM, at 2-106. Make sure to include your **name and recitation number** in your homework! The numbers of the sections and exercises refer to " Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.".

Lecture 4:

- **Read:** book sections 2.5 and 2.6.
- **Work:** book section 2.5 (exercises 6, 8 and 23) and 2.6 (6, 10, 13, and 20).

Lecture 5:

- **Read:** book section 2.7.
- **Work:** book section 2.7 (exercises 4, 12, 13, 17, 36 and 40).

Lecture 6:

- **Read:** book section 3.1.
- **Work:** book section 3.1 (exercises 5, 10, 18, 23 and 24).

Challenge Problem

This is about the important $n \times n$ tridiagonal matrices K_n and T_n with $-1, 2-1$ on all the middle rows. The $(1,1)$ entry of T_n is 1 when the $(1,1)$ entry of K_n is 2:

$$K_4 = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}; \qquad T_4 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix};$$

a) Write MATLAB commands that will create K_5 and T_5 (the *toeplitz* command is useful, or *diag(v,k)* puts a vector v on diagonal k).

b) Factor K_4 and T_4 in the form
$LDL^T =$ (lower triangular)(diagonal pivots)(lower triangular).
You can do it by hand, or by using *lu* or *chol* commands.

c) What is L^{-1} for K_4 and T_4?

d) What are the inverse matrices K_4^{-1} and T_4^{-1}?

e) Find a formula (You don't need to prove it, if you don't want) for the (i, j) entry of T_n^{-1} (the easy one) and K_n^{-1} (a little harder, notice each row of K_4^{-1} and K_5^{-1} on and above the diagonal).

Please staple your solution as first page of your homework.