18.06 - Spring 2004 - Problem Set 4

March 4, 2004

This problem set on lectures $11 - 14$ is due Wednesday (March 10th), at 4 PM, at 2-106. Make sure to include your **name and recitation number** in your homework! The numbers of the sections and exercises refer to " Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.".

Lecture 11:

- **Read:** book section 3.6.
- **Work:** book section 3.6 (exercises 4, 25, 26, 28 and 29)

Lecture 12:

- **Read:** book section 8.2.
- **Work:** book section 8.2 (exercises 8, 11 and 17).

Lecture 13:

- **Read:** book section 4.1.
- **Work:** book section 4.1 (exercises 3, 6, 7, 26 and 28).

Lecture 14:

- **Read:** book section 4.2.
- **Work:** book section 4.2 (exercises 4, 13, 17, 19, 25, 27 and 29).

Challenge Problem Let A be a 3×3 matrix. Prove that if A^3 is not the zero matrix, then A^n is not zero for all positive integer n.

Hint: Divide the problem into cases, according to the possible ranks this matrix can have. You may use without proving the very nice fact that any matrix B can be written as the sum of rank one matrices: $B = u_1 v_1^T + \ldots + u_k v_k^T$, where k is the rank of the matrix B.

Please staple your solution as first page of your homework.