18.06 – Professor Strang – Quiz 1 – Solutions

March 5, 2004

1. (a) Doing elimination at this matrix gives you that the last row is $(0, 0, c - 14)$, so $c = 14$.

   Another way to do it is to try to write the third column (row) as a linear combination of the 2 others. Clearly $c = 14$ does the job. Of course, computing the determinant and setting it equal to zero also works.

   (b) If the student did elimination already in the first part, they will observe that $x_p = (6, 2, 0)^T$.

   By inspection of the columns, the right hand side equals 6 times the first column plus 2 times the second – one particular solution is $x_p = (6, 2, 0)^T$. For the general solution to $Ax = 0$, we can do elimination in A, and solve the new system

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 20 \end{bmatrix}$$

   getting that $N(A)$ is spanned by $(-4, -2, 1)^T$.
   Hence, $x_c = x_p + z(-4, -2, 1)^T$.

   (c) Column Picture: Three column vectors spanning a plane, and the right hand side belongs to the same plane.

   Row Picture: 3 planes intersecting on a line.

2. (a) Row 3 must be a linear combination of the first two rows.

   (b) By row reducing A, we get that $a = 4$ and $b = 5$.

   (c) $N(A) = N(R)$, hence $N(A)$ is the two-dimensional space spanned by $(-2, 1, 0, 0)$ and $(3, 0, 2, -1)$.

3. **Note:** The grader will have to check each student example to see if z is not a combination of u v and w and THEN to see the dimensions of $C(A)$ $N(A)$.

   u, v can be any vectors, w a linear combination of them, and z not contained in the subspace spanned by u and v.

(b) Depends on the example – if u, v are l.i., dim $C(A) = 2$, dim $N(A) = 1$; if not, dim $C(A) = 1$, dim $N(A) = 2$.

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4. For this question, Prof. Strang, in lecture, found L directly from the multipliers 2, 2 and 0 (that go below the 1's in the diagonal of L, and U is found by elimination.

   Some students may find the elimination matrix E first, and then invert it (as follows).

   Performing elimination on A gives you, already in the first step,

$$U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

   Hence, $EA = U$, where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix},$$

   so in order to get $A = LU$, just need to invert E, which is easy:

$$L = E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix},$$