Denote by $C_{ij}$ the $n \times n$ matrix having a 1 in the $(i,j)$ entry, and 0’s everywhere else. In terms of these matrices we may write $E_{ij} = I - \ell_{ij}C_{ij}$ and $E^{-1}_{ij} = I + \ell_{ij}C_{ij}$. Thus we have

$$ME^{-1}_{ij} = M(I + \ell_{ij}C_{ij}) = M + \ell_{ij}MC_{ij}$$

The matrix $MC_{ij}$ has all columns different from the $j$–th consisting entirely of 0’s. The $j$–th column of $MC_{ij}$ is simply the $i$–th column of $M$. Since the matrix $E_{ij}$ is a lower triangular matrix we have $i > j$, and since $M$ is only filled in up to column $j$, the $i$–th column of $M$ has exactly one 1 in the $i$–th row and 0’s everywhere else. Therefore the matrix $MC_{ij}$ has exactly one non-zero entry in position $(i,j)$ and this entry is a 1, i.e. $MC_{ij} = C_{ij}$.

We conclude that $ME^{-1}_{ij} = M + \ell_{ij}C_{ij} = N$, as required.

The effect of right multiplication by $E^{-1}_{ij}$ on a matrix $M$ is to leave the columns of $M$ different from the $j$–th one unchanged, and replacing the $j$–th column of $M$ by the sum of the $j$–th column of $M$ with $\ell_{ij}$ times the $i$–th column of $M$. 
