Challenge Problem 1

Suppose $R$ (an $m \times n$ matrix) is in row reduced echelon form $\begin{pmatrix} I & F \\ 0 & 0 \end{pmatrix}$, with $r$ nonzero rows and first $r$ pivot columns.

a) Describe the column space and nullspace of $R$.
b) Do the same for the $m \times 2n$ matrix $B = \begin{pmatrix} R & R \end{pmatrix}$.
c) Do the same for the $2m \times n$ matrix $C = \begin{pmatrix} R \\ R \end{pmatrix}$.
d) Finally, do the same for the $2m \times 2n$ matrix $D = \begin{pmatrix} R & R \\ R & R \end{pmatrix}$.

Challenge Problem 2

a) Suppose that $A$ is a $3 \times 3$ matrix. What relation is there between the nullspace of $A$ and the nullspace of $A^2$? How about the nullspace of $A^3$?
b) The set of polynomials of degree at most four in the variable $x$ is a vector space. What is the nullspace of $\frac{d^2}{dx^2}$? What is the nullspace of $\left(\frac{d^2}{dx^2}\right)^2$?