This problem set is due Wednesday (March 16th), at 4 PM, in 2-106. Make sure to PRINT your name, recitation number and instructor on your homework! Please staple your MATLAB solutions as first pages of your homework.

Lecture 15:

- **Read**: book section 4.3.
- **Work**: book section 4.3 (exercises 4, 9, 12, 26 and 27)

Lecture 16:

- **Read**: book section 4.4.

Lecture 17:

- **Read**: book section 5.1.
- **Work**: book section 5.1 (exercises 3, 12, 15, 28 and 34).

**Challenge Problem with MATLAB**

The command `a = ones(n, 1)` produces an $n \times 1$ matrix of 1’s.

The command `l = (1:n)'` produces the vector $(1, 2, \ldots, n)$, transposed to a column by `'`.

The command `s = l.^2` produces the vector $1^2, 2^2, \ldots, n^2$, because the dot means “a component at a time.”

This problem looks for the line $y = c + dt$ closest to the parabola $y = t^2$ on the interval $t = 0$ to $t = 1$.

1. Find the best line by calculus, not MATLAB. Choose $c$ and $d$ to minimize

   $$E(c, d) := \int_0^1 (c + dt - t^2)^2 dt$$

2. With $n = 10$, choose $C$ and $D$ to give the line $y = C + Dt$ that is closest to $t^2$ at the points $t = \frac{1}{10}, \frac{2}{10}, \ldots, 1$ (in the vector $1/10$). The unsolvable equations $AX = b$ (use least squares) are

   $$\begin{bmatrix} a & 1/n \\ C \\ D \end{bmatrix} = s/n^2$$

   Find the best $C$ and $D$ and the errors $c - C$ and $d - D$.

3. Repeat for $n = 20$. (Notice how $1/n$ and $s/n^2$ end at 1, like the calculus problem.)

   Are the differences $c - C$ and $d - D$ smaller for $n = 20$ and by approximately what factor?