Your PRINTED name is: ____________________________

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1 (17 pts.) If the output vectors from Gram-Schmidt are

\[ q_1 = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad \text{and} \quad q_2 = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \]

describe all possible input vectors \( a_1 \) and \( a_2 \).
2 (15 pts.) If $a$ and $b$ are nonzero vectors in $\mathbb{R}^n$, what number $x$ minimizes the squared length $\|b - xa\|^2$?
Find the projection $p$ of the vector $b = (1, 2, 6)$ onto the plane $x + y + z = 0$ in $\mathbb{R}^3$. (You may want to find a basis for this 2-dimensional subspace, even an orthogonal basis.)
Find the determinants of $A$ and $A^{-1}$ and the $(1,2)$ entry of $A^{-1}$ if

$$A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 2 & 1 & 3 \\
1 & 3 & 1 & 7
\end{bmatrix}.$$
5  (17 pts.)  By recursion or cofactors or otherwise(!) compute the determinant of this 5 by 5 circulant matrix $C$:

$$C = \begin{bmatrix}
2 & -1 & 0 & 0 & -1 \\
-1 & 2 & -1 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 \\
0 & 0 & -1 & 2 & -1 \\
-1 & 0 & 0 & -1 & 2 \\
\end{bmatrix}$$
Suppose $P_1$ is the projection matrix onto the 1-dimensional subspace spanned by the first column of $A$. Suppose $P_2$ is the projection matrix onto the 2-dimensional column space of $A$. After thinking a little, compute the product $P_2P_1$.

\[
A = \begin{bmatrix}
1 & 0 \\
2 & 1 \\
0 & 1 \\
1 & 2 
\end{bmatrix}.
\]