1. Section 6.1, Problem 2

*Answer:* $A$ has $\lambda_1 = -1$ and $\lambda_2 = 5$ with eigenvectors $x_1 = (-2, 1)$ and $x_2 = (1, 1)$. The matrix $A + I$ has the same eigenvectors with eigenvalues increased by 1: $\lambda_1 = 0$ and $\lambda_2 = 6$.

2. Section 6.1, Problem 12

*Answer:* $P$ has $\lambda = 1, 0, 1$ with eigenvectors $(1, 2, 0), (2, -1, 0), (0, 0, 1)$. $P^{100} = P$ so $P^{100}$ has the same eigenvalues and eigenvectors. An eigenvector with no zero components is $(1, 2, 0) + (0, 0, 1) = (1, 2, 1)$ which has $\lambda = 1$.

3. Section 6.1, Problem 22

*Answer:* $A$ and $A^T$ have the same eigenvalues because $\det(A - \lambda I) = \det(A - \lambda I)^T = \det(A^T - (\lambda I)^T) = \det(A^T - \lambda I)$. 

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \text{ and } A^T = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \text{ have different eigenvectors.}$$

4. Section 6.1, Problem 28

*Answer:* $\text{rank}(A) = 1$, with $\lambda = 0, 0, 4$. $\text{rank}(C) = 2$, with $\lambda = 0, 0, 2, 2$. 

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5. Section 6.2, Problem 3

Answer:

\[ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}. \]

6. Section 6.2, Problem 15

Answer: (No explanation necessary.)

a) True; all eigenvalues are non-zero.

b) False; may have 2 or 3 independent eigenvectors.

c) False; may have 2 or 3 independent eigenvectors.

7. Section 6.2, Problem 22

Answer:

\[ A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}. \]

\[ A^k = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3^k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3^k + 1 & 3^k - 1 \\ 3^k - 1 & 3^k + 1 \end{bmatrix}. \]

8. Section 6.2, Problem 29

Answer: If \( A \) has columns \( x_1, \ldots, x_n \), then \( A^2 = A \) means that \( Ax_i = x_i \) for every \( x_i \). All vectors in the column space are eigenvectors with \( \lambda = 1 \). Always the nullspace has \( \lambda = 0 \).
9. Section 8.3, Problem 12

Answer: 2, 3, 5 as the last row makes $A$ Markov and symmetric. When $A$ is Markov and symmetric, each row adds to 1 so $(1, 1, 1, 1)$ is an eigenvector of $A$.

10. Section 10.2, Problem 2

Answer:

$$A^H A = \begin{bmatrix} 2 & 0 & 1 + i \\ 0 & 2 & 1 + i \\ 1 - i & 1 - i & 2 \end{bmatrix} \quad \text{and} \quad A^H A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

are both Hermitian matrices.

11. Section 10.2, Problem 8

Answer: $P$ is orthogonal, invertible, unitary and factorizable into $QR$. 