Problem 1 Wednesday 2/07

Consider the following system of equations:

\[
\begin{align*}
    x + 3y + 2z &= 6 \\
    2x + 5y + 4z &= 1 \\
    3x + 8y + 6z &= 7 \\
\end{align*}
\]

What do you notice about the equations?
The first two planes intersect in a line. What do you know about that line and the third plane? How many solutions does the system have?

Problem 2 Wednesday 2/07

(a) Find a matrix \( A \) such that

\[
\begin{bmatrix}
    2 \\
    0 \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
    1 \\
    3 \\
\end{bmatrix}
\]

(b) What is

\[
\begin{bmatrix}
    3 \\
    3 \\
\end{bmatrix}
\]

Problem 3 Wednesday 2/07

Do problem 26 of section 2.1 in your book.

Problem 4 Wednesday 2/07

Let’s practice using Matlab to check that in general \( AB \) and \( BA \) are not equal. (Hint: you can type \texttt{diary} at the beginning of your session to save a transcript.)

Let’s start with matrices of different sizes. Let \( A=\text{ones}(3,2) \) and \( B=\text{ones}(2,3) \) (that is, the 3-by-2 and 2-by-3 matrices with all entries equal to 1). Compute \( AB \) and \( BA \). What are their sizes?

Now, let’s multiply to 3-by-3 matrices. Let \( C=\begin{bmatrix} a & b & c; d & e & f; g & h & i \end{bmatrix} \), where \( a \ldots i \) are nine of your favorite numbers. \textit{Now let the computer pick one:} \( D=\text{rand}(3,3) \) gives us a random 3-by-3 matrix. What are \( CD \) and \( DC \)? Are they equal?

Problem 5 Friday 2/09

Write examples of systems \( A\vec{x} = \vec{b} \) where \( A \) is a 3-by-3 matrix and:

1. the three planes meet in a common line
2. in the row picture, all three planes are parallel but distinct
3. the intersection of the first two planes does not intersect the third plane
4. \( \vec{b} \) is not a linear combination of the columns of \( A \).
5. in the column picture, \( \vec{b} \) is a multiple of the second column of \( A \).
Problem 6 Friday 2/09
Answer the following questions for the systems in problem 5:
(a) How many solutions does each have? Describe the shape (point, line, ...) of each solution set.
(b) Reduce each by elimination (you need not back-substitute) and check your answer.

Problem 7 Friday 2/09
Solve the following system by elimination and back substitution:

\[
\begin{align*}
2x + 3y + z &= 0 \\
x - 2y - z &= -3 \\
x + y + 2z &= 3
\end{align*}
\]
Write down the elimination matrices \( E_{21}, E_{31}, E_{32} \) you used.

Problem 8 Monday 2/12
Consider the matrices
\[
A = \begin{bmatrix} 5 & -3 & -9 \\ 2 & 4 & -1 \\ -1 & 7 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ -3 & 3 \end{bmatrix}.
\]
(a) Find \( AB \) and \( AC \).
(b) Do you notice anything special? Why does this tell you \( A \) is not invertible?

Problem 9 Monday 2/12
Do problem 13 of section 2.4 in your book.

Problem 10 Monday 2/12
Do problem 7 of section 2.5 in your book.