18.06 Problem Set 5  
Due Wednesday, March 21, 2007 at 4:00 p.m. in 2-106

**Problem 1** Wednesday 3/14  
Do problem 10 of section 4.3 in your book.

**Problem 2** Wednesday 3/14  
Do problem 17 of section 4.3 in your book.

**Problem 3** Wednesday 3/14  
Find a function of the form $f(t) = C\sin(t) + D\cos(t)$ that approximates the three points $(0,0)$, $(\pi/2, 2)$ and $(\pi, 1)$. In other words, find coefficients $C$ and $D$ such that the error $|f(0) - 0|^2 + |f(\pi/2) - 2|^2 + |f(\pi) - 1|^2$ is as small as possible.

**Problem 4** Wednesday 3/14  
The MATLAB command `a=ones(n,1)` produces an $n$-by-1 matrix of ones. The command `r=(1:n)'` gives the vector $(1, 2, \ldots, n)$ transposed to a column by '. The command `s=r.^3` gives the column vector $(1^3, 2^3, \ldots, n^3)$, because the dots mean "a component at a time."

The purpose of this problem is to find the line $y = c + dt$ closest to the cubic function $y = t^3$ on the interval $t = 0$ to $t = 1$.

(a) Find the best line using calculus, not MATLAB. Choose $c$ and $d$ to minimize

$$E(c, d) := \int_0^1 (c + dt - t^3)^2 dt$$

(Hint: find $E(c, d)$ in terms of $c$ and $d$, and use any method learned in 18.02 to minimize this.)

(b) With $n = 15$, choose $C$ and $D$ to give the line $y = C + Dt$ that is closest to $t^3$ at the points $t = \frac{1}{15}, \frac{2}{15}, \ldots, 1$. Use MATLAB to do least squares in order to find $C$ and $D$, and the differences $c - C$ and $d - D$.

(Hint: Set up the equations you want to solve. Your equations (in matrix form) should involve the vectors `a`, `r/n` and `s/n.^3`).

(c) Repeat for $n = 30$. (Notice how `r/n` and `s/n.^3` end at 1). Are the differences $c - C$ and $d - D$ smaller for $n = 30$? By what factor?

**Problem 5** Friday 3/16  
Consider in $\mathbb{R}^4$ the subspace given by $F = \{(x, y, z, w) : -x + y + 2z - w = 0\}$.

(a) Give a basis for $F$.

(b) Use Gram-Schmidt to transform your basis into an orthonormal basis.

(c) What is the distance between the point $(1, 3, 1, 1)$ and (the closest point to) $F$?
**Problem 6** Friday 3/16

Do problem 6 of section 4.4 in your book.

**Problem 7** Friday 3/16

(a) Find a 3-by-3 orthogonal matrix $A$ such that

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

(b) How many matrices $A$ are there that satisfy these conditions?

**Problem 8** Monday 3/19

Do problem 9 of section 5.1 in your book.

In the following problems explain how you calculated the determinants.

**Problem 9** Monday 3/19

Calculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}.$$

**Problem 10** Monday 3/19

(a) Calculate the determinants of the following “almost upper-triangular” matrices:

$A_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},$

$A_3 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix},$

$A_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix},$

$A_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$

(b) Can you figure out how to continue the sequence of matrices $A_2, A_3, A_4, A_5, \ldots$ and calculate $det(A_n)$ for any $n$?