Your PRINTED name is: ________________________

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1) M 2 2-131 A. Ritter 2-085 2-1192 afr
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9) T 12 26-142 P. Buchak 2-093 3-1198 pmb
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12) T 1 26-168 P. McNamara 2-314 4-1459 petermc
13) T 2 2-132 B. Lehmann 2-089 2-1195 lehmann
14) T 2 26-168 P. McNamara 2-314 4-1459 petermc
1 (18 pts.) Start with an invertible 3 by 3 matrix $A$. Construct $B$ by subtracting 4 times row 1 of $A$ from row 3. **How do you find $B^{-1}$ from $A^{-1}$?** You can answer in matrix notation, but *you must also answer in words*—what happens to the columns and rows?
2 (24 pts.) Elimination on $A$ leads to $U$:

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{leads to} \quad Ux = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$ 

(a) Factor the first matrix $A$ into $A = LU$ and also into $A = LDL^T$.

(b) Find the inverse of $A$ by Gauss-Jordan elimination on $AA^{-1} = I$ or by inverting $L$ and $D$ and $L^T$.

(c) If $D$ is diagonal, show that $LDL^T$ is a symmetric matrix for every matrix $L$ (square or rectangular).
3 (30 pts.) Suppose the nonzero vectors $a_1, a_2, a_3$ point in different directions in $\mathbb{R}^3$ but
\[3a_1 + 2a_2 + a_3 = \text{zero vector} .\]
The matrix $A$ has those vectors $a_1, a_2, a_3$ in its columns.

(a) Describe the nullspace of $A$ (all $x$ with $Ax = 0$).

(b) Which are the pivot columns of $A$?

(c) I want to show that all 3 by 3 matrices with
\[\text{(\star)} \quad 3(\text{column 1}) + 2(\text{column 2}) + (\text{column 3}) = \text{zero vector} \]
form a subspace $S$ of the space $M$ of 3 by 3 matrices. Now the zero matrix is certainly included.

Suppose $B$ and $C$ are matrices whose columns have this property (\star).

To show that we have a subspace, we have to prove that every linear combination of $B$ and $C$ (finish sentence).

Go ahead and prove that.
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4 (28 pts.) Start with this 2 by 4 matrix:

\[
A = \begin{bmatrix}
2 & 3 & 1 & -1 \\
6 & 9 & 3 & -2
\end{bmatrix}
\]

(a) Find all special solutions to \(Ax = 0\) and describe the nullspace of \(A\).

(b) Find the complete solution—meaning all solutions \((x_1, x_2, x_3, x_4)\)—to

\[
Ax = \begin{bmatrix}
2x_1 + 3x_2 + x_3 - x_4 \\
6x_1 + 9x_2 + 3x_3 - 2x_4
\end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.
\]

(c) When an \(m\) by \(n\) matrix \(A\) has rank \(r = m\), the system \(Ax = b\) can be solved for which \(b\) (best answer)? How many special solutions to \(Ax = 0\)?
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