Your PRINTED name is: _____________________________

Please circle your recitation:

(R01) M2 2-314 Qian Lin
(R02) M3 2-314 Qian Lin
(R03) T11 2-251 Martina Balagovic
(R04) T11 2-229 Inna Zakharevich
(R05) T12 2-251 Martina Balagovic
(R06) T12 2-090 Ben Harris
(R07) T1 2-284 Roman Bezrukavnikov
(R08) T1 2-310 Nick Rozenblyum
(R09) T2 2-284 Roman Bezrukavnikov

Grading

1

2

3

Total:
Your classmate, Nyarlathotep, performed the usual elimination steps to convert $A$ to echelon form $U$, obtaining:

$$U = \begin{pmatrix}
1 & 4 & -1 & 3 \\
0 & 2 & 2 & -6 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}.$$  

(a) Find a set of vectors spanning the nullspace $N(A)$.

(b) If $Uy = \begin{pmatrix}
9 \\
-12 \\
0 \\
\end{pmatrix}$, find the complete solution $y$ (i.e. describe all possible solutions $y$).

(c) Nyarla gave you a matrix

$$L = \begin{pmatrix}
1 & 0 & 0 \\
2 & 1 & 0 \\
-1 & 3 & 1 \\
\end{pmatrix}$$

and told you that $A = LU$. Describe the complete sequence of elimination steps that Nyarla performed, assuming that she did elimination in the usual way starting with the first column and eliminating downwards. That is, Nyarla first subtracted _____ times the first row from the second row, then subtracted _____ times the first row from the third row, then subtracted ________________________________.

(Be careful about signs: adding a multiple of a row is the same as subtracting a negative multiple of that row.)

(d) If $Ax = \begin{pmatrix}
0 \\
2 \\
6 \\
\end{pmatrix}$, then $Ux = _____.}
Which of the following (if any) are subspaces? For any that are not a subspace, give an example of how they violate a property of subspaces.

(I) Given some $3 \times 5$ matrix $A$ with full row rank, the set of all solutions to $Ax = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(II) All vectors $x$ with $x^T y = 0$ and $x^T z = 0$ for some given vectors $y$ and $z$.

(III) All $3 \times 5$ matrices with $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in their column space.

(IV) All $5 \times 3$ matrices with $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in their nullspace.

(V) All vectors $x$ with $\|x - y\| = \|y\|$ for some given fixed vector $y \neq 0$. 
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3 (20 pts.) \(A\) is a matrix with a nullspace \(N(A)\) spanned by the following three vectors:

\[
\begin{pmatrix}
1 \\
2 \\
-1 \\
3
\end{pmatrix},
\begin{pmatrix}
0 \\
1 \\
1 \\
4
\end{pmatrix},
\begin{pmatrix}
-1 \\
-1 \\
3 \\
1
\end{pmatrix}.
\]

(\(\alpha\)) Give a matrix \(B\) such that its column space \(C(B)\) is the same as \(N(A)\).

(There is more than one correct answer.) [Thus, any vector \(y\) in the nullspace of \(A\) satisfies \(Bu = y\) for some \(u\).]

(\(\beta\)) Give a different possible answer to (\(\alpha\)): another \(B\) with \(C(B) = N(A)\).

(\(\gamma\)) For some vector \(b\), you are told that a particular solution to \(Ax = b\)

\[
x_p = \begin{pmatrix}
1 \\
2 \\
3 \\
4
\end{pmatrix}.
\]

Now, your classmate Zarkon tells you that a second solution is:

\[
x_Z = \begin{pmatrix}
1 \\
1 \\
3 \\
0
\end{pmatrix},
\]

while your other classmate Hastur tells you “No, Zarkon’s solution can’t be right, but here’s a second solution that is correct:”

\[
x_H = \begin{pmatrix}
1 \\
1 \\
3 \\
1
\end{pmatrix}.
\]

Is Zarkon’s solution correct, or Hastur’s solution, or are both correct?

(Hint: what should be true of \(x - x_p\) if \(x\) is a valid solution?)
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