18.06 Quiz 2 Professor Strang April 6, 2011

Your PRINTED name is ______________________ **1.**

Your Recitation Instructor (and time) is ______________________ **2.**

Instructors: (Pires)(Hezari)(Sheridan)(Yoo) **3.**

Please show enough work so we can see your method and give due credit.

1. **(8 pts. each)** Suppose a_1 and a_2 are orthogonal unit vectors in R^5.

   (a) What are the requirements on a matrix P to be a projection matrix? Verify that $P = a_1 a_1^T + a_2 a_2^T$ satisfies those requirements.

   (b) If a_3 is in R^5, what combination of a_1 and a_2 is closest to a_3?

   (c) Find a combination c of a_1, a_2, a_3 that is perpendicular to a_1 and a_2. *If possible, choose* $c \neq 0$. Describe all cases when $c = 0$ is the only possibility.

   (d) Show that a_1 and a_2 and c are eigenvectors of P (*if* $c \neq 0$) and find their eigenvalues.

2. **(7 pts. each)**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 0 & 9 & 10 \\ 0 & 0 & 11 & 12 \end{bmatrix}.$$

(a) Find all nonzero terms in the big formula $\det A = \sum \pm a_{1\alpha}\, a_{2\beta}\, a_{3\gamma}\, a_{4\delta}$ and combine them to compute $\det A$.

(b) Find all the pivots of A.

(c) Find the cofactors C_{11}, C_{12}, C_{13}, C_{14} of row 1 of A.

(d) Find column 1 of A^{-1}.

3. **(8 pts. each)** Suppose A is a 2 by 2 matrix and $Ax = x$ and $Ay = -y$ ($x \neq 0$ and $y \neq 0$).

(a) (Reverse engineering) What is the polynomial $p(\lambda) = \det(A - \lambda I)$?

(b) If you know that the first column of A is $(2, 1)$, find the second column:

$$A = \begin{bmatrix} 2 & ? \\ 1 & ? \end{bmatrix}.$$

(c) For that matrix in part (b), find an invertible S and a diagonal matrix Λ so that $A = S\Lambda S^{-1}$.

(d) Compute A^{101}. (If you don't solve parts (b) -(c), use the description of A at the start. In all questions **show enough work** so we can see your method and give due credit.)

(e) If $Ax = x$ and $Ay = -y$ (with $x \neq 0$ and $y \neq 0$) prove that x and y are *independent.*

Start of a proof: Suppose $z = cx + dy = 0$. Then $Az =$ (follow from here.)