18.06 Quiz 3 Professor Strang May 6, 2011

Your PRINTED name is ______________________________ 1.

Your Recitation Instructor (and time) is ______________________________ 2.

Instructors: (Hezari)(Pires)(Sheridan)(Yoo) 3.

Please show enough work so we can see your method and give due credit.

1. (a) Find two eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}.$$

(b) Express any vector $u_0 = \begin{bmatrix} a \\ b \end{bmatrix}$ as a combination of the eigenvectors.

(c) What is the solution $u(t)$ to $\frac{du}{dt} = Au$ starting from $u(0) = u_0$?

(d) Find a formula $u_k =$ __________ for the solution to $u_{k+1} = Au_k$ which starts from that vector u_0. **Set $k = -1$ to find $A^{-1}u_0$.**

2. This problem is about the matrix

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ 0 & \sqrt{2} \end{bmatrix}.$$

(a) Find all eigenvectors of A. Exactly why is it impossible to diagonalize A in the form $A = S\Lambda S^{-1}$?

(b) Find the matrices U, Σ, V^T in the Singular Value Decomposition $A = U\, \Sigma\, V^T$.
Tell me *two orthogonal vectors* v_1, v_2 in the plane so that Av_1 and Av_2 are also orthogonal.

(c) Find a matrix B that is similar to A (but different from A).
Show that A and B meet the requirement to be similar *(what is it?)*.

3. Suppose A is a real m by n matrix.

(a) Prove that the symmetric matrix $A^T A$ has the property $x^T(A^T A)x \geq 0$ for every vector x in R^n. Explain each step in your reason.

(b) According to part (a), the matrix $A^T A$ is positive semidefinite at least — and possibly positive definite. Under what condition on A is $A^T A$ positive definite?

(c) If $m < n$ prove that $A^T A$ is *not* positive definite.