2.6 #13 We can guess the following decomposition:

\[
L = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    1 & 1 & 0 & 0 \\
    1 & 1 & 1 & 0 \\
    1 & 1 & 1 & 1 \\
\end{bmatrix}
\quad U = \begin{bmatrix}
    a & a & a & a \\
    0 & b-a & b-a & b-a \\
    0 & 0 & c-b & c-b \\
    0 & 0 & 0 & d-c \\
\end{bmatrix}
\]

2.6, #24 Suppose \( A = LU \) for an invertible \( A \); then \( A_k = L_k U_k \), where \( A_k \) denotes the \( k \times k \) matrix formed by looking at the entries which lie in both the first \( k \) rows and the first \( k \) columns (and similarly for \( L_k \) and \( U_k \)). But clearly \( L_k \) and \( U_k \) are invertible, since they have non-zero diagonal entries (since \( L \) and \( U \) are invertible); hence for each \( 1 \leq k \leq n \), \( A_k \) is invertible, i.e. has non-zero determinant.

2.7 #3 Since \((AB)^T = B^T A^T\):
\((Ax)^T y = x^T A^T y = x^T (A^T y)\)

2.7 #40 (a) Consider the equation \( Q^T Q = I \); the \((i,i)\)-th entry is 1; expanding this tells us that the \( i \)-th column has norm 1.
(b) Consider the \((i,j)\)-th entry of the equation \( Q^T Q = I \); when \( i,j \) are unequal, the \((i,j)\)-th entry is 0; expanding this means that the \( i \)-th column is perpendicular to the \( j \)-th column.
(c) \[
\begin{bmatrix}
    \cos(t) & -\sin(t) \\
    \sin(t) & \cos(t) \\
\end{bmatrix}
\]

3.1 #5 (a) For instance, the subspace consisting only of scalar multiples of \( A \).
(b) Yes, since \( I = A - B \).
(c) Consider the subspace consisting only of the following matrices:
\[
\begin{bmatrix}
    0 & x \\
    0 & 0 \\
\end{bmatrix}
\]

3.1 #10 Only (a), (d), (e).

3.1 #15 (a) line (but it could be a plane)
(b) point (but it could be a line)
(c) Say \( x,y \) in \( X \cap Y \). Then: \( x+y \) is in \( X \), and \( x+y \) is in \( Y \), so \( x+y \) is in \( X \cap Y \); similarly \( cx \) is in \( X \) and \( cx \) is in \( Y \) so \( cx \) is in \( X \cap Y \).

3.1 #20 (a) \( b_2 = 2b_1, b_1 + b_3 = 0 \) (b) \( b_1 + b_3 = 0 \)

3.1 #23 unless the extra column lies in the span of the columns of \( A \).

The column space gets bigger in the following example:
\[
\begin{bmatrix}
    1 \\
    0 \\
\end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix}
    1 \\
    0 \\
\end{bmatrix}
\]

The column space stays the same in this example: \([1] \rightarrow [1 2]\)
#24 $A = [1]$, $B = [0]$, $AB = [0]$ (the column space of $AB$ is zero, while the column space of $A$ is 1-dimensional)

**Q9** 4 x 4 permutation matrices $\iff$ permutations of $\{1,2,3,4\}$

the permutation matrix will be symmetric $\iff$ the permutation is an involution

there are 10 involutions:
1, (12), (13), (14), (23), (24), (34), (12)(34), (14)(23), (13)(24); the corresponding matrices are:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}

**Q10**

\[
\begin{bmatrix}
1 & 2 & 4 \\
2 & 4 & 8 \\
4 & 8 & 16 \\
\end{bmatrix}
\]