18.06 (Spring 14) Problem Set 1

This problem set is due Thursday, Feb 13, 2014 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. This homework has 10 questions worth 100 points in total. Please WRITE NEATLY. You may discuss with others (and your TA), but you must turn in your own writing.

There is normally a MATLAB part. Go to lms.mitx.mit.edu, and follow the instructions on course website. The online homework this week is optional. It is an introduction to MATLAB. If you are already comfortable with MATLAB, you do not need to do it. However, it is worth getting signed into and set up with the MITx website, so please sign in and take a look.

1. In Lecture 1, Prof. Strang drew the cone (infinite triangle) that comes from all combinations $cv + dw$ with $c \geq 0$ and $d \geq 0$. Which c and d would give that triangle cut off by a top line from v to w? Which c and d give the parallelogram that starts with sides v and w ?

2. The length of v is $||v|| = \sqrt{v' * v} = \sqrt{v_1^2 + ... + v_n^2}$. The dot product $v' * w$ equals $||v||||w||$ times the cosine of the angle between v and w. If $||v|| = 3$ and $||w|| = 5$, what are the smallest and largest possible values of the dot product $v' * w$ and of $||v - w||$?

3. The column vectors $u = (1, 1, 2)$, $v = (1, 2, 3)$ and $w = (3, 5, 8)$ are in a plane because w is what combination of u and v? Find two combinations of u, v, w that produce $b = (0, 0, 0)$ and two combinations that produce $b = (1, -1, c)$. What is the only possible number c that gives a vector on the plane?

   These problems come from Introduction to Linear Algebra (4th edition)

4. Problem 19 on page 42 (Section 2.1: elimination matrices)

5. Problem 35 on page 44: Which row exchanges of a 9 by 9 Sudoku matrix will produce another Sudoku matrix (with the numbers 1 to 9 in every column and row and 3 by 3 block)? Which BLOCK row exchanges will produce another Sudoku matrix? Since N rows can be ordered in $N!$ ways, how many different Sudoku matrices are there?

6. Problem 8 on page 52 (breakdown of elimination, Section 2.2)

7. Problem 11 on page 53 (Section 2.2, never exactly two solutions)

8. Problem 26 on page 54 (matrices with given row and column sums)

9. Problem 32 on page 55 (100 equations in 100 unknowns: singular case)

10. Problem 29 on page 66 (Section 2.3, reducing the 4 by 4 Pascal matrix)