18.06 (Spring 14) Problem Set 6

This problem set is due Thursday, April 3, 2014 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. This homework has 8 questions worth 80 points in total. Please WRITE NEATLY. You may discuss with others (and your TA), but you must turn in your own writing.

1. Suppose every vector in $\mathbf{R}^2$ is multiplied by the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$.

   (a) Describe the shape that comes from multiplying every vector in thesquare centered at $(0,0)$ with side length equal to 2 by A.

   (b) What is the area of that shape ?

2. Suppose you have **any** shape with area 1 in the xy plane. If you multiply every vector in that shape by the same A as above, then the area is multiplied by the determinant of A. Why is this ?

3. Find the determinant of the 3×3 matrix A whose (i,j) entry is $3^{|i-j|}$. Find the inverse of A using the cofactor matrix and the determinant ($A^{-1} = C^T / \det A$).

4. (p. 252, problem 11) Suppose that $CD = -DC$ and find the flaw in this reasoning: Taking determinants gives $|C|\,|D| = -\,|D|\,|C|$. Therefore $|C| = 0$ or $|D| = 0$. One or both of the matrices must be singular.

5. Prove that every orthogonal matrix Q has determinant $+1$ or -1.

6. Find the determinant of $I + M$, if M is the rank one matrix $M = vv^T$, where v is a column vector $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

7. Suppose A_n is the $n \times n$ symmetric tridiagonal matrix with a subdiagonal of 1's, a main diagonal of 3's, and a superdiagonal of 1's. By cofactors of row 1, connect the determinant of A_n to the determinants of A_{n-1} and A_{n-2}.

8. (a) Find orthonormal vectors q_1, q_2, q_3 such that q_1 and q_2 span the column space of $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 2 & -2 \end{bmatrix}$. Use Gram-Schmidt.

   (b) Which of the four fundamental subspaces for A will contain q_3?

   (c) Solve $Ax = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$ by least squares.

9. MATLAB problems: Please go to lms.mitx.mit.edu to finish this part.