18.06 (Spring 14) Problem Set 9

This problem set is due Thursday, May 1, 2014 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. This homework has 7 questions worth 70 points in total. Please WRITE NEATLY. You may discuss with others (and your TA), but you must turn in your own writing.

1. Problem 6.6.17 P.362. True or false, with a good reason:
   (a) A symmetric matrix can't be similar to a nonsymmetric matrix.
   (b) An invertible matrix can't be similar to a singular matrix.
   (c) A can't be similar to $-A$ unless $A = 0$.
   (d) A can't be similar to $A + I$.

2. Problem 6.6.18 P.362. If B is invertible, prove that AB is similar to BA. They have the same eigenvalues.

3. Problem 6.7.1 P.371. Find the eigenvalues and unit eigenvectors v_1, v_2 of $A^T A$. Then find $u_1 = Av_1/\sigma_1$:
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, A^T A = \begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix}, AA^T = \begin{bmatrix} 5 & 15 \\ 15 & 45 \end{bmatrix}.$$
   Verify that u_1 is a unit eigenvector of AA^T. Complete the matrices U, Σ, V.
$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T.$$

4. Problem 6.7.14 P.372. Suppose A is invertible (with $\sigma_1 > \sigma_2 > 0$). Change A by as small a matrix as possible to produce a singular matrix A_0. Hint: U and V do not change:
   From $A = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$, find the nearest A_0.

5. Problem 10.2.3 P.506. Solve $Az = 0$ to find a vector in the nullspace of
$$A = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}.$$
   Show that z is orthogonal to the columns of A^H. Show that z is not orthogonal to the columns of A^T. The good row space is no longer $\mathbf{C}(A^T)$. Now it is $\mathbf{C}(A^H)$.

6. Problem 10.2.6 P.507. True or false (give a reason if true or a counterexample if false):
   (a) If A is a real matrix then $A + iI$ is invertible.
   (b) If A is a Hermitian matrix then $A + iI$ is invertible.
   (c) If A is a unitary matrix then $A + iI$ is invertible.

7. Problem 10.2.19 P.508. The functions e^{-ix} and e^{ix} are orthogonal on the interval $0 \le x \le 2\pi$ because their inner product is $\int_0^{2\pi}$ _____________ $= 0$.