18.06 Exam 1 review

1. Fill the blanks: for an $n \times n$ invertible matrix A, the column space $C(A) =$______, the null space $N(A) =$______, the pivot columns are ______, $R = rref(A)$ equals ______, and the solution to $A\mathbf{x} = \mathbf{b}$ is ______.

2. Answer to the same question as in Problem 1, when $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \end{bmatrix}$. What are the special solutions to $A\mathbf{x} = \mathbf{0}$?

3. Answer to the same question as in Problem 1, when $A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$. What are the special solutions to $A\mathbf{x} = \mathbf{0}$?

4. If E is a square and invertible matrix,
   (a) How is $C(EA)$ related to $C(A)$?
   (b) How is $N(EA)$ related to $N(A)$?
   (c) How is $rref(EA)$ related to $rref(A)$?

5. If A is a 5×6 matrix and $R = rref(A)$,
   (a) Why are there nonzero solutions to $A\mathbf{x} = \mathbf{0}$?
   (b) How is $C(\begin{bmatrix} A & A \end{bmatrix})$ related to $C(A)$? (Note that $\begin{bmatrix} A & A \end{bmatrix}$ is a 5×12 matrix.)
   (c) There are at least __ special solutions to $\begin{bmatrix} A & A \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$.

6. If P is a permutation matrix, explain why $P^N = I$ holds for some positive integer N.

7. (a) If A and B are 4×4 and AB is invertible, show that A is invertible.
   (b) A 5×4 matrix times a 4×5 matrix cannot produce an invertible 5×5 matrix. Why not?

8. Here are 8 equivalent statements (plus 2 more that involve $A^T A$ — coming soon).
   (1) The columns of A are independent
   (2) The rows of A span $\mathbf{R}^n$
   (3) The rank of A is n : "full column rank"
   (4) All the columns of A are pivot columns so $R = \begin{bmatrix} I \\ 0 \end{bmatrix}$
   (5) The nullspace $\mathbf{N}(A)$ contains only the zero vector
   (6) The row space $\mathbf{C}(A^T)$ is all of $\mathbf{R}^n$
   (7) The columns of A are a basis for its column space
   (8) If $Ax = Ay$ then $x = y$ (uniqueness of solutions to $Ax = b$)
   (9) The matrix $A^T A$ is invertible (and symmetric positive definite)
   (10) A has a left-inverse $B = (A^T A)^{-1} A^T$, with $BA = I$

   Can you find 8 (or 10) parallel statements, all equivalent to this statement 1?

   (1) The rows of A are independent.

9. Spring 2014, Exam 1, Problem 2

10. Spring 2014, Exam 1, Problem 3
11. Fall 2012, Exam 1, Problem 1
12. 3.5.26
13. 3.6.16
14. A good final practice set is to try Exam1 from Fall 2014. Remember you only have 50min to do it.