18.06 Problem Set 4

Due Thursday, March 12, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has several questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online part and you can choose whether to do it (self-graded) on MITx with MATLAB or in Julia. Follow the instructions on course website.

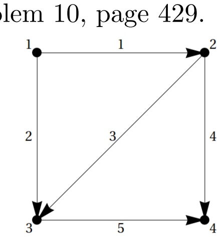
Problem 1. Section 3.6, Problem 28, page 194. Find the ranks of the 8 by 8 checkerboard matrix B and the chess matrix C:

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} r & n & b & q & k & b & n & r \\ p & p & p & p & p & p & p & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p & p & p & p & p & p & p & p \\ r & n & b & q & k & b & n & r \end{bmatrix}$$

The numbers r, n, b, q, k, p are all different. Find bases for the row space and left nullspace of B and C.

Challenge problem for bonus points: Find a basis for the nullspace of C. Analyze both cases: $p = 0$ and $p \neq 0$.

Problem 2. Section 8.2, Problem 10, page 429.



Write down the 5 by 4 incidence matrix A for the square graph with two loops. Reduce A to its echelon form U. The three nonzero rows give the incidence matrix for what graph? You found one tree in the square graph—find the other seven trees.

Problem 3. Section 8.2, Problem 9, page 429.

For the square graph above, find two requirements on the b's for the five differences $x_2 - x_1$, $x_3 - x_1$, $x_3 - x_2$, $x_4 - x_2$, $x_4 - x_3$ to equal b_1, b_2, b_3, b_4, b_5. You have found Kirchhoff's ___________ around the two ___________ in the graph.

Problem 4. Section 8.2, Problem 17, page 430.

Suppose A is a 12 by 9 incidence matrix from a connected (but unknown) graph.

(a) How many columns of A are independent?
(b) What condition on f makes it possible to solve $A^T y = f$?
(c) The diagonal entries of $A^T A$ give the number of edges into each node. What is the sum of those diagonal entries?

Problem 5. Section 4.1, Problem 28, page 205.

Why is each of these statements false?

(a) $(1, 1, 1)$ is perpendicular to $(1, 1, -2)$ so the planes $x+y+z = 0$ and $x+y-2z = 0$ are orthogonal subspaces.
(b) The subspace spanned by $(1, 1, 0, 0, 0)$ and $(0, 0, 0, 1, 1)$ is the orthogonal complement of the subspace spanned by $(1, -1, 0, 0, 0)$ and $(2, -2, 3, 4, -4)$.
(c) Two subspaces that meet only at the zero vector are orthogonal.

Problem 6. Section 4.1, Problem 30, page 205.

Suppose A is 3 by 4 and B is 4 by 5 and $AB = 0$. So $\boldsymbol{N}(A)$ contains $\boldsymbol{C}(B)$. Prove from the dimensions of $\boldsymbol{N}(A)$ and $\boldsymbol{C}(B)$ that $\operatorname{rank}(A) + \operatorname{rank}(B) \leq 4$.