18.06 Exam the First

17 February 2016

NAME: ____

GRADING	
1. _______	/20
2. _______	/20
3. _______	/20
4. _______	/20
5. _______	/20
TOTAL	/100

1. Yay or nay

For each of the following collection of vectors in $\mathbf{R}^3$, answer YES or NO: are they linearly independent? (You do *not* have to justify your answer.)

(a) $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

(b) $\left\{ \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \right\}$

(c) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 5 \end{pmatrix} \right\}$

(d) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 17 \\ 0 \\ 0 \end{pmatrix} \right\}$

(e) $\left\{ \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix} \right\}$

(f) $\left\{ \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix}, \begin{pmatrix} 8 \\ 8 \\ 5 \end{pmatrix} \right\}$

2. Construct

In $\mathbf{R}^3$, find three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ such that the angle between any pair of them is $\pi/3$.

3. Do not compute

How many solutions does the following system of 100 linear equations in variables $x_1, x_2, \dots, x_{100}$ have?

$$
\begin{aligned}
x_1 + x_2 + \cdots + x_{98} + x_{99} &= 1 \\
x_1 + x_2 + \cdots + x_{98} + x_{100} &= 2 \\
&\vdots \\
x_1 + x_3 + \cdots + x_{99} + x_{100} &= 99 \\
x_2 + \cdots + x_{99} + x_{100} &= 100
\end{aligned}
$$

4. Intersectionalism

What's the angle between the following two lines in $\mathbf{R}^9$?

$$\lambda_1(t) = (t, -t, t, -t, t, -t, t, -t, t)$$

and

$$\lambda_2(t) = (2t, 2t, 2t, 2t, 2t, 2t, 2t, 2t, 2t)$$

5. The pentachoron

In $\mathbf{R}^2$, we had the equilateral triangle, in $\mathbf{R}^3$, we had the regular tetrahedron, and in $\mathbf{R}^4$, we have the *regular pentachoron*, which is made up of 5 tetrahedra. It has 5 vertices:

$$\left(1, 1, 1, -\tfrac{1}{\sqrt{5}}\right),$$
$$\left(1, -1, -1, -\tfrac{1}{\sqrt{5}}\right),$$
$$\left(-1, 1, -1, -\tfrac{1}{\sqrt{5}}\right),$$
$$\left(-1, -1, 1, -\tfrac{1}{\sqrt{5}}\right),$$
$$\left(0, 0, 0, \sqrt{5} - \tfrac{1}{\sqrt{5}}\right)$$

If there were such a thing as 4-dimensional methane, it would have a "carbonoid" atom bonded to 5 "hydrogenoid" atoms, arranged in a regular pentachoron whose center would be the "carbonoid" atom. What would be the angle between the bonds in 4-dimensional methane?