18.06 Exam IV: Eigenstuff

29 April 2016

STATE YOUR NAME: _____

R01	10–11	Sauer–Ayala
R02	10–11	Carpentier
R03	11–12	Sauer–Ayala
R04	11–12	Carpentier
R05	12–13	Hopkins
R06	12–13	Anno
R07	13–14	Hopkins
R08	13–14	Anno
R09	14–15	Fei
R10	14–15	Knizel
R11	15–16	Knizel

CIRCLE YOUR RECITATION:

GRADING

1. _____ /20

2. _____ /20

3. _____ /20

4. _____ /20

5. _____ /20

TOTAL

/100

1. Yah or neh?

For each of the following sentences, indicate whether they are true or false. (No need to justify your answer.)

(a) If A is an $n \times n$ matrix in which one row is a multiple of another, then $\det(A) = 0$.

(b) If A is a real $n \times n$ matrix, then $\det(A^2) \geq 0$.

(c) If A is an $n \times n$ matrix with n distinct real eigenvalues, then A is diagonalizable over $\mathbf{R}$.

(d) There are matrices that are diagonalizable over $\mathbf{C}$ but not diagonalizable over $\mathbf{R}$.

(e) The characteristic polynomial of an $n \times n$ matrix with complex entries has degree n.

2. Determine ant

Compute

$$\det \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

3. Dudley Eigenvalue, DDS

Compute the eigenvalues and eigenspaces of

$$\begin{pmatrix} 1 & 9 \\ 4 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 9 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Are these matrices diagonalizable over $\mathbf{R}$? Over $\mathbf{C}$?

4. COMPLICATIONS

Compute the eigenvalues and eigenspaces of

$$\begin{pmatrix} 3 & -2 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

Is this matrix diagonalizable over $\mathbf{R}$? Over $\mathbf{C}$?

5. Vade mecum

Compute the eigenvalues and eigenspaces of

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -4 \end{pmatrix}.$$