1 18.06 pset 3

Due Wednesday March 1 at 11am.

Note: Exam 1 is on Friday March 3 in room 50-340.

1.1 Problem 1

Suppose that you solve $AX = B$ with

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and find that $X$ is

$$X = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{pmatrix}$$

1.1.1 (a)

What is $A^{-1}$?

(You should not have to apply brute-force Gaussian elimination to invert any matrices, nor should you use Julia in this part. You should be able to show how to do this quickly by hand.)

(This is not because we care about hand calculation per se, but rather because it is useful to be able to recognize and exploit special structure in matrices, and to understand the relationship between solving systems of right-hand-sides and finding $A^{-1}$.)

1.1.2 (b)

Evaluate a simple expression to check your answer from (a) by brute-force calculation in Julia.

For example, you can compute $B^{-1}X^{-1}$ by `inv(B) * inv(X)` in Julia. There should be some simple product of matrices or matrix inverses that gives $A^{-1}$. Figure it out!

```julia
In [ ]: # here are the matrices B and X in Julia form
    B = [1 1 1 1
         0 2 2 2
         0 0 1 1
         0 0 0 1]
    X = [1 1 0 1
         0 0 1 0
         1 3 1 0
         2 0 0 1]

In [ ]: inv(B) * inv(X)  ## FIX THIS: change to an expression that will give A^{-1}, and evaluate
```
1.2 Problem 2
Consider the vector space $\mathcal{M}$ of $m \times m$ real-valued matrices for some $m$, say $m = 4$. True or false (and provide a counter-example if false).

1. The symmetric matrices in $\mathcal{M}$ are a subspace (matrices with $A^T = A$).
2. The “skew-symmetric (also called”antisymmetric“) matrices (those with $A^T = -A$) in $\mathcal{M}$ are a subspace.
3. The invertible matrices in $\mathcal{M}$ are a subspace.
4. The singular matrices in $\mathcal{M}$ are a subspace.

1.3 Problem 3
(Strang, section 3.2, problem 22.) If $AB = 0$ then the column space of $B$ is contained in the _____ of $A$. Why?

1.4 Problem 4
(Strang, section 3.2, problem 29.) If $A$ is $4 \times 4$ and invertible, what is the nullspace of the $4 \times 8$ matrix $B = (A \ A)$?

1.5 Problem 5
(Strang, section 3.2, problem 23.) The reduced-row echelon form $R$ of a $3 \times 3$ matrix with randomly chosen entries is almost sure to be _______. What $R$ is virtually certain if the random matrix is $4 \times 3$?

1.6 Problem 6
(Strang, section 3.2, problem 58.) Suppose $R$ is $m \times n$ of rank $r$, with pivot columns first:

$$R = \begin{pmatrix} I & F \\ 0 & 0 \end{pmatrix}$$

where $I$ is an identity matrix and 0 denotes a block of zeros.

1. What are the shapes of those four blocks of $R$?
2. Find a right-inverse matrix $B$ such that $RB = I$ if $r = m$ (the zero blocks are gone).
3. What is the reduced-row echelon form of $R^T$?
4. What is the reduced-row echelon form of $R^T R$?

(In the last four parts, indicate both blocks like $I$ or 0 and their shapes.)