18.06 Professor Edelman Quiz 1 March 1, 2019

Your name is: ______________________________

(We need your name on every page for gradescope.)

(Exam ends at 11:55am. We have been requested to vacate the room immediately.)

Please circle your recitation:

(1)	T 10	24-307	S. Makarova
(2)	T 10	4-261	Z. Remscrim
(3)	T 11	24-307	S. Makarova
(4)	T 11	4-261	C. Hewett
(5)	T 12	4-261	C. Hewett
(6)	T 12	2-105	A. Ahn
(7)	T 1	4-149	A. Ahn
(8)	T 1	2-136	S. Turton
(9)	T 2	2-136	K. Choi
(10)	T 3	2-136	K. Choi

Important Instructions: We will be using Gradescope. Please write on one side only of a page. If you need extra pages, please write continued in the box, and on the extra pages clearly label with problem number and letter.

Name: ______________________________

1 (20 pts.)

The full SVD of

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 8 & 2 \\ 5 & 12 & 2 \end{pmatrix}$$

is numerically computed with Julia to be

$$U = \begin{pmatrix} -0.203600 & -0.585801 & -0.784465 \\ -0.543021 & -0.599144 & 0.588348 \\ -0.814662 & 0.545769 & -0.196116 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 6.136942826453964 & & \\ & 0.7740001393771697 & \\ & & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.365991 & 0.446524 & 0.816497 \\ -0.912869 & -0.00172137 & -0.408248 \\ -0.180887 & -0.89477 & 0.408248 \end{pmatrix}$$

1.(a) (5 pts.) The rank of this matrix is

Name: ____________________

1. (b) (5 pts.) The column space is a linear combination of some vectors found in the svd. Circle these vectors.

$$U = \begin{pmatrix} -0.203600 & -0.585801 & -0.784465 \\ -0.543021 & -0.599144 & 0.588348 \\ -0.814662 & 0.545769 & -0.196116 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 6.136942826453964 & & \\ & 0.7740001393771697 & \\ & & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.365991 & 0.446524 & 0.816497 \\ -0.912869 & -0.00172137 & -0.408248 \\ -0.180887 & -0.89477 & 0.408248 \end{pmatrix}$$

Name: ____________________

1. (c) (5 pts.) Circle the numbers that would figure in the best rank 1 approximation to A.

$$U = \begin{pmatrix} -0.203600 & -0.585801 & -0.784465 \\ -0.543021 & -0.599144 & 0.588348 \\ -0.814662 & 0.545769 & -0.196116 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 6.136942826453964 & & \\ & 0.7740001393771697 & \\ & & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.365991 & 0.446524 & 0.816497 \\ -0.912869 & -0.00172137 & -0.408248 \\ -0.180887 & -0.89477 & 0.408248 \end{pmatrix}$$

Name: ______________________________

1. (d) (5 pts.) Circle the non-zero numbers that would figure in the compact (rank-r) svd

$$U = \begin{pmatrix} -0.203600 & -0.585801 & -0.784465 \\ -0.543021 & -0.599144 & 0.588348 \\ -0.814662 & 0.545769 & -0.196116 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 6.136942826453964 & & \\ & 0.7740001393771697 & \\ & & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.365991 & 0.446524 & 0.816497 \\ -0.912869 & -0.00172137 & -0.408248 \\ -0.180887 & -0.89477 & 0.408248 \end{pmatrix}$$

Name: ____________________________

2. (20 pts.)

The matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is known to have a decomposition of the form

$$A = \begin{pmatrix} p & q \\ 0 & r \end{pmatrix} \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix},$$

where $r > 0$. Find r in terms of possibly a, b, c, d.

Name: ______________________________

3. (25 pts.) Am I a vector space? Briefly explain why or why not. (Remember the zero must be in the vector space, and all real linear combinations must be in the vector space.)

a. The vectors in R^3 where $x^2 + y^2 + z^2 \leq 1$.

b. All of R^3 except those vectors along the x-axis with $x > 0$. (This means $(x, 0, 0)^T$ is excluded if $x > 0$.)

c. All 2x3 matrices whose 6 elements sum to 6.

d. All 3x3 rank 1 matrices and the 3x3 zero matrix.

e. All functions of two variables $f(x, y)$ of the form $f(x, y) = ax^2 + bxy + c$ such that $f(18.06, 2019) = 0$.

Name: ________________________________

4. (15 pts.) Count the parameters

How many parameters are there in an $n \times n$ "anti-symmetric" matrix? An anti-symmetric matrix has $A^T = -A$, for example when $n = 1, 2, 3$, anti-symmetric matrices look like

$$\begin{pmatrix} 0 \end{pmatrix}, \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}, \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

Your answer should be a simple function of n .

Name: ____________________________________

5. (20 pts.) The nullspace and QR

Let

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

(a) (10pts.) We will inform you that the nullspace of the matrix A is a line. You should be able to tell us exactly which line this is without any difficult computations. Please describe the line.

(b) (10 pts.) The QR factorization of A^T happens to be very easy to find without any difficult computations. Write down the QR factorization of A^T.

Name: ____________________________________

6. (Extra Credit 5 pts.) This problem is only worth five points. Some of you may see the answer right away, but others may not see it at all. We do not recommend looking at this problem unless you have extra time, as the five points may not be worth the time lost. Suppose an $n \times 2$ matrix A is written as QR, where Q is tall-skinny orthogonal and is also $n \times 2$, and

$$R = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

What is the norm of the second column of A? Explain briefly.