Your name is: _________________________________  Grading  

Please circle your recitation:

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1 Find the eigenvalues and eigenvectors of these matrices:

(a) (10) Projection $P = \frac{aa^T}{a^Ta}$ with $a = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

(b) (10) Rotation $Q = \begin{bmatrix} .6 & -.8 \\ .8 & .6 \end{bmatrix}$

(c) (8) Reflection $R = 2P - I$
2 (a) (10) Find the eigenvalues $\lambda_1$, $\lambda_2$, $\lambda_3$ (NOT the eigenvectors $x_1$, $x_2$, $x_3$) of this Markov matrix:

$$A = \begin{bmatrix} .6 & .6 & 0 \\ .2 & .2 & .2 \\ .2 & .2 & .8 \end{bmatrix}$$

(b) (10) Suppose $u_0$ is the sum $x_1 + x_2 + x_3$ of the three eigenvectors that you didn’t compute. What is $A^n u_0$?

(c) (4) As $n \to \infty$ what is the limit of $A^n u_0$?
(a) (2 each) Suppose $M$ is any invertible matrix. Circle all the properties of a matrix $A$ that remain the same for $M^{-1}AM$:

- same rank
- same nullspace
- same determinant
- real eigenvalues
- orthonormal eigenvectors
- symmetric positive definiteness

(b) (2 each) This is a similar question but now $Q$ is an orthonormal matrix. Circle the properties of $A$ that remain the same for $Q^{-1}AQ$:

- same column space
- $A^k$ approaches zero as $k$ increases
- orthonormal eigenvectors
- symmetric positive definiteness
- projection matrix
(a) (3 each) Suppose the 5 by 4 matrix $A$ has independent columns. What is the most information you can give about

- the eigenvalues of $A^T A$: 
- the eigenvectors of $A^T A$: 
- the determinant of $A^T A$: 

(b) (9) Find the singular value decomposition (SVD) for this matrix:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix}.$$

(c) (8) When the input basis is $v_1, \ldots, v_n$ and the output basis is $w_1, \ldots, w_n$ and the matrix of the linear transformation $T$ using these bases is the identity matrix, what is $T(v_1 + v_2)$?