Your PRINTED name is: 

Please circle your recitation:

1. T 10 26-328 D. Kubrak
2. T 11 26-328 D. Kubrak
3. T 12 4-159 P.B. Alvarez
4. T 1 4-149 P.B. Alvarez
5. T 2 4-149 E. Belmont
6. T 3 4-261 J. Wang

Grading

Total:

Note: We are not planning to use gradescope for this exam.
1 \textbf{(25 pts.)} Find the QR decomposition (Q is $4 \times 2$, R is $2 \times 2$ upper triangular) of

\[
A = \begin{pmatrix}
1 & a \\
1 & b \\
1 & c \\
1 & d
\end{pmatrix}
\]

in terms of $\mu = \frac{a + b + c + d}{4}$, the mean of the second column and the elements of $A$. 
2 (20 pts.) An experimenter has data in the form of pairs \((x_i, y_i)\) for \(i = 1, \ldots, n\) where the \(x_i\) are distinct and positive. Given the matrix

\[
A = \begin{pmatrix}
\sin(x_1) & e^{x_1} & \sqrt{x_1} \\
\sin(x_2) & e^{x_2} & \sqrt{x_2} \\
\vdots & \vdots & \vdots \\
\sin(x_n) & e^{x_n} & \sqrt{x_n}
\end{pmatrix},
\]

suggest a method for computing the best fit function of the form \(f(x) = C \sin(x) + D e^x + E \sqrt{x}\) through the \(n\) points. In what precise sense is your answer a best fit?
3 (20 pts.)
A form of the singular value decomposition of a rank r, \( m \times n \) matrix \( A \) is \( U\Sigma V^T \) where \( \Sigma \) is square \( r \) by \( r \) with positive diagonal entries, \( U \) is \( m \times r \) and \( V \) is \( n \times r \). Write down projection matrices for the four fundamental subspaces of \( A \), in terms of one of \( U \), \( \Sigma \), or \( V \) in each expression. Be sure to clearly identify which fundamental subspace of \( A \) goes with which projection matrix.
4 (35 pts.)
Let $d(A)$ be a scalar function of $3 \times 2$ matrices $A$ with the following properties:

$\alpha$) If you interchange the two columns of $A$, $d(A)$ flips sign.

$\beta$) $d(A)$ is linear in each of the columns of $A$.

$\gamma$) $d(A)$ is non-zero for at least one $3 \times 2$ $A$.

a. (5 pts.) What is $d(2A)$ in terms of $d(A)$?

b. (10 pts.) Give an example $d(A)$ that satisfies the three requirements of this question.
c. (10 pts.) We recall that the determinant of square matrices is linear in each column and each row of the square matrix. Can property $\beta$ be extended to rows and columns of $3 \times 2$ matrices $A$ to create a $d(A)$ with the three requirements of this question? If yes, give an example, if not, why not?

d. (10 pts.) If we discard property $\gamma$ to allow the “zero” function, the set of all functions $d(A)$ satisfying $\alpha$ and $\beta$ form a three dimensional vector space. Describe explicitly this vector space of functions in terms of the elements of $A$. 
Your Initials: ____________

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Your Initials: __________

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