Lecture 2 (Easy cases): Q&A

Gaussian integral

How do you derive $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$*?*

For that story, see Section 2.1.2 in the notes.

In $\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx$*, why don't the easy cases involve* x*?*

The x has been integrated away because of the limits. So only α remains after doing the integral and putting in limits, and so the only quantity that can vary is α. Think of it like:

$$\text{left side} = \int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx.$$

The goal is to guess the left side, which is a function just of α (not x).

Pyramid

Could two pyramids make a cube (imagine a plane slicing the cube diagonally)?

I don't think so – each pyramid would have $h = b$, where b is the side length of the cube, and therefore volume $b^3/3$. Two of these pyramids would have a combined volume $2b^3/3$ rather than b^3, which is the volume of the cube. Maybe three pyramids, with a clever arrangement, could make a cube?

How can $V = ha^2$ *when* $b = 0$*? Isn't* a *constrained to be less than* b*?*

You're right: The truncated pyramid was constructed so that $a < b$. Once you construct it, however, flip it on its head but still use a as the side length of the top surface and b as the side length of the bottom surface. Then you have a pyramid with $b < a$.

General

Do rocks mimic math or vice versa? Life is constrained within math, not the other way round.

I'm not sure. I think that if rocks behaved differently, we'd invent different mathematics. For example, classical electromagnetism is a linear theory: Electric fields from different charges simply add to one another. So, a lot of mathematics was invented to study linear systems (e.g. linear algebra, 6.003).

The causation does not work in reverse. Having linear mathematics does not mean that electromagnetism is actually linear. Indeed, the linearity is only approximate: Quantum electrodynamics, because of pair creation, is a nonlinear theory. And a lot of mathematics (e.g. Feynman diagrams) was invented to handle its nonlinearities.

But, the question belongs to a huge area in which it would be foolish to claim certainty, for who can know for certain the principles on which the world is constructed? So I'm somewhat confident of my answer given above, but not certain!

Are easy cases always obvious? Are there tricks to find them?

There are a couple rules of thumb. First, look for the extreme cases. If a quantity can vary between 0 and ∞, usually 0 and ∞ are useful extreme cases. Second, look for symmetries among the quantities. In the truncated-pyramid problem, for example, a and b are symmetric: Switching a and b turns the pyramid on its head but does not change the volume. Then $a = b$ is likely to be another useful easy case.

What is the take-home message?

The principle of easy cases: A correct answer works in all cases, including the easy cases. So, use easy cases to check and to guess answers.

Why aren't we taught these methods in other classes?

I wish I understood the reason!

I heard there might be a textbook this year.

Hope springs eternal. I wanted to turn last year's notes into the textbook for this year, and they are almost ready. Now I plan to finish turning this year's notes into the textbook, soon after IAP. So hopefully there will be a textbook next year.