18.325: The Mathematics of Finite Random Matrices

Professor Alan Edelman

Handout #1, Tuesday, February 1, 2005 — Course Outline

Summary. This is a course on the mathematics and applications of *finite* random matrices. Our aim is to touch upon various branches of the study of finite random matrices—a consequence is that we will end up lingering on some areas longer than others. Our hope is that this course will confer:

- some familiarity with several of the main thrusts of work in random matrices—sufficient to give you some context for formulating and seeking known solutions to applications in engineering and physics;
- sufficient background and facility to let you read current research publications in the area of finite random matrix theory;
- a set of tools, both analytical and computational, for the analysis of new random matrices that arise in new problems you may encounter.

Course page. `http://web.mit.edu/~18.325/www/`

Lecturer. Alan Edelman, `edelman@math.mit.edu`. URL: `http://www-math.mit.edu/~edelman/`.

TA. Raj Rao, `raj@mit.edu`.

Mailing List. You can add yourself to the class mailing list 18.325 using `mailmaint` on Athena.

Content. The goal is for the course is, paradoxically, to be broad *as well as* deep. Our plan (unlikely to survive contact with the trials and tribulations of an actual semester here at MIT) is to touch upon the following broad areas while attempting to uncover deep insights into the underlying mechanisms that unify these areas. This is a tentative list of topics that *might* be covered in the course; We will select material adaptively based on our background, interests, and rate of progress. If you are interested in some other topics, please let us know and we'd be happy to accomodate your interests.

Matrix Calculus Matrix Jacobians, Computing Jacobians of 2 x 2 matrices. Jacobians of simple matrix factorizations.

Wedge Products Notation to "simplify" computation of Matrix Jacobians for complicated matrix factorizations.

Classical Random Matrix Ensembles The Wishart, Gaussian and Jacobi ensembles, their joint eigenvalue densities, Haar distributed orthogonal/unitary matrices, Stiefel/Grassman manifold.

Matrix Integrals Classical orthogonal polynomials, the Cauchy-Binet theorem, correlation functions.

Equilibrium Measure. The Hermite, Laguerre and Jacobi orthogonal polynomials. Interpretation of the limiting distribution as the equilibrium measure of (univariate) orthogonal polynomials. Applications to physics.

Fredholm Determinants. Tracy-Widom Distribution. Eigenvalue spacings and the Riemann-Zeta Hypothesis.

Jack Polynomials. Multivariate orthogonal polynomials. Combinatorial aspects. Connections to random matrices.

Applications. Wireless Communications, Statistical Physics.

Prerequisites. We assume that the reader has had an undergraduate course in Linear Algebra (18.06) or its equivalent and some exposure to probability (6.041 or 6.042 are more than sufficient).

Requirements. Final project

Mid-Term Project. You will be asked to read a paper on a topic of interest to you that involves random matrix theory and present it via some mixture of the following perspectives:

- Write a description *of greater clarity* than the original publication, or
- Devise an improved solution to the problem under consideration, and write up your improvement (with appropriate discussion of the original solution).
- Implement the result in `MATLAB` in order to study its performance in practice. Considerations include choice of random matrix result, design of good tests, interpretation of results, and design and analysis of heuristics for improving performance in practice.

Semester project. The semester project can be an extension of the mid-term project if it sustains your interest. Otherwise, you will be asked to come up with some insights into a random matrix problem that is of interest to you. Alternately you can use state-of-the-art tools developed recently to plot some eigenvalue statistics and compare it to predictions. There will be more details on this as we go through the semester.

In this spirit, and since this is an advanced graduate class on a very active research area, the grading will be like any other advanced graduate class on a cutting-edge topic. Please contact one of us for any clarifications.

Textbooks. There are no textbooks covering a majority portion of the material we will be studying in this course. We will be giving out course readers during the semester to help you study the material before the lectures. There will be reserch papers handed out in class and posted on the website.

Guest Speakers.

We will have guest speakers that will help give us their own perspective on their research involving random matrices. While these speakers will appear as part of the Applied Math Colloquium series, attendance is strongly encouraged.