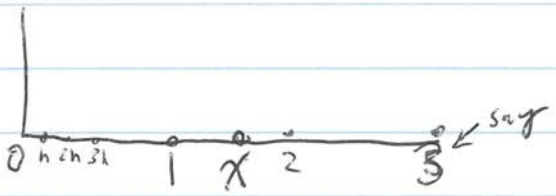
 Feb 19, 2013

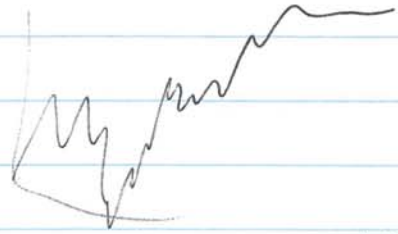
1. Brownian Motion $\boxed{p192}$

Remember if $x_i \sim N(0, \sigma_i^2)$, $i=1,\dots,n$
then $\sum x_i \sim N(0, \sum \sigma_i^2)$

or if $x_i = \sigma_i G$ (G ind randn's)
$\sum x_i = \sqrt{\sum \sigma_i^2}\, G$ "Add in Pythagorean way"



h (=.01 say)
x = [h : h : 3]
dW = randn(length(x), 1) * sqrt(h)
W = cumsum(dw)
plot(x, W)



Let x be any interior position

$w(h) \sim N(0,h)$
$w(2h) \sim N(0,2h)$
$w(x) \sim N(0,x)$

$$W(x) \sim N(0, x)$$

People write $h = dx$ or Δx
so $dW \sim N(0, dx) = G\sqrt{dx}$
$W'(x) = \frac{dW}{dx} \approx \frac{dW}{h}$ discrete time Noise Process

$W'(x)$ has mean 0 + variance $\frac{1}{h}$

$$E\, W'(x) W'(y) = \delta(x-y)$$

Analog of $x = \text{randn}(n,1)$

$$E(x(i) \cdot x(j)) = \delta_{ij}$$

2. Let x be any scalar random variable
+

$$x + dW + dW + dW + dW + \dots$$

starts out x
eventually Gaussian

Standard Method for interpolating

$$A(x) = \begin{pmatrix} w(x) & \cdots & w(x) \\ \vdots & & \\ w(x) & & w(x) \end{pmatrix}$$

$$S(x) = \left(\frac{A(x) + A(x)^T}{\sqrt{x}} \right) \quad \text{All GOE's}$$

Brownian Motion GOE

Let's you make a "path" of GOES

Theorem Let T_n be the random sym Tridiagonal Matrix

$$\frac{1}{\sqrt{n\beta}}\begin{pmatrix} G\sqrt{2} & \chi_{\beta(n-1)} & & \\ \chi_{\beta(n-1)} & \ddots & \ddots & \\ & \ddots & \ddots & \chi_\beta \\ & & \chi_\beta & G\sqrt{2} \end{pmatrix}$$

This tridiagonal has the same eigenvalue distribution as Gaussian Ensembles but reveals interesting mathematics & requires $2n-1$ storage & $O(n^2)$ computation (vs $O(n^2)$ & $O(n^3)$)

Interesting Math

1. General β obvious (& works)
2. $\lim_{n\to\infty}$ gives a stochastic operator

Proof of Theorem

Householder etc