18.338 Eigenvalues of Random Matrices

Problem Set 3

Due Date: Wed Feb. 27, 2013

Reading and Notes

Read chapter *6* and *10* of the class notes. Please again give your feedback especially high level style and where things did not make sense, in addition to spelling or technical errors

Homework

Do at least four out of the following problems (Computational/Mathematical problems are denoted as C/M. Exercise with numbers and pages are from the class notes. There is a super problem (8) that counts for the whole homework, meaning either choose 4 out of the first 7 problems or just do problem 8.

1. (C) Investigate heavy-tailed distributions (We don't know what you will see!) Please refer to Wikipedia for specific examples of heavy-tailed distributions. One example is the Weibull distribution (In MATLAB, `wblrnd(A, B, m, n)`).

```
n = 500;
A = wblrnd(1, 0.5, n); % generate n by n matrix A with heavy-tailed entries
A = (A - 2)/sqrt(20); % normalize to have mean 0 and std 1
S = (A + A')/sqrt(2*n); % form the Wigner matrix
```

   Do a Monte Carlo example as in Code 1.2 in the notes. Investigate smaller and larger n, describe whether the histogram "wants" to be a semicircle or "wants" to be different at different scale for n (consider small n, medium n and large n behaviors). For those who are interested in some of the mathematical results, see Professor Guionnet's lecture notes (http://www.umpa.ens-lyon.fr/~aguionne/LectureNotes18177.pdf, section 2.3 perhaps around page 16).

2. (M) Exercise 6.4 (p68)

3. (M) Exercise 6.5 (p68)

4. (M) Exercise 10.3 (p133)

5. (M) Exercise 10.6 (p134)

6. (C) Code 5.6 uses the formula at the bottom of page 56 to calculate the exact eigenvalue density for finite GUE ($\beta = 2$ Hermite) matrices. Alternatively, one can use (5.4) (the Christoffel-Darboux relationship). This problem asks you to change Hermite to Laguerre.

   For the monte carlo experiment, one needs to take the squares of the `svd` of `randn(m,n)` for $m \geqslant n$.

   For the theoretical density, you may use code 5.2 to calculate the Laguerre polynomials at x of degree 0 to $n-1$ with parameter a. Make pretty picture like Figure 5.2.

   For extra credit, do the same for Jacobi using code 5.3. The $\beta = 1$ Jacobi ensemble is on page 1 of notes (http://web.mit.edu/18.338/www/2012s/handouts/lec4.pdf).

7. (C) In Julia or MATLAB evolve Dysonian Brownian motion either through a dreict simulation

   `lambda = lambda + sqrt(h) * randn(n,1) + h*(1/r potentials)`

   or by computing eigenvalues

   `Matrix = Matrix + sqrt(h) * GOE`

   Probably impossible: extra credit, devise a test to reverse engineer whether it was GOE or Dysonian brownian motion from code output.

8. (C*) This is a complete problem worth the entire homework. Write a MATLAB or Julia code that computes the GOE version of the finite level densities. There is a mathematica code (see the attached notes) based on symbolic integration. It should be doable based on Code 5.1.