

■ Source: The Transition to Chaos: Conservative Classical Systems and Quantum Manifestations, L. Reichl, 2nd edition, page 205

Some basic definitions

```
In[20]:= randn[n_] := Table[RandomReal[NormalDistribution[0, 1]], {r, 1, n}, {c, 1, n}];
symmetrize[mat_] := 
$$\frac{\text{mat} + \text{ConjugateTranspose}[\text{mat}]}{2}$$
;
randGOE[n_] := symmetrize[randn[n]];
randGUE[n_] := symmetrize[randn[n] + I randn[n]];
realeigs[mat_] := Map[Re, Eigenvalues[mat]];
```

The functions  $\phi$ , basically the Hermite polynomials multiplied by the Gaussian weight. For GOE eigenvalues, we will need the derivatives and integrals of these functions as well.

```
In[40]:= 
$$\phi[n_, x_] := \left(\sqrt{\pi} 2^n n!\right)^{-1/2} \text{Exp}\left[-\frac{1}{2} x^2\right] \text{HermiteH}[n, x];$$


$$\phi\text{deriv}[n_, x_] := \text{D}[\phi[n, y], y] /. y \rightarrow x;$$


$$\phi\text{int}[n_, x_] := \text{Integrate}[\phi[n, t], \{t, 0, x\}];$$

```

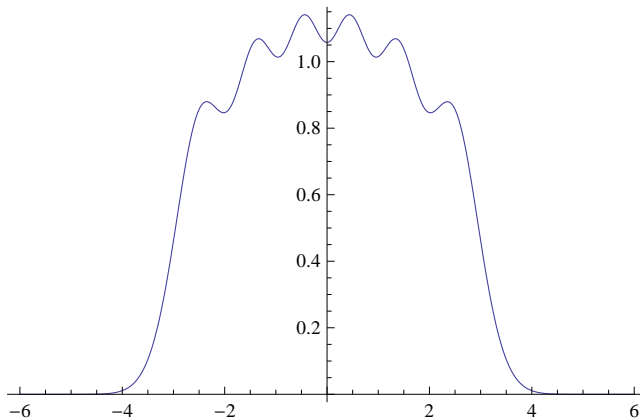
The formula for the eigenvalue density of the GUE is just  $\sum_{k=0}^{N-1} \phi_k(x)^2$ .

```
In[2]:= eigDensityGUE[N_, x_] := Sum[\phi[k, x]^2, {k, 0, N-1}];
```

The theoretical density for N=6...

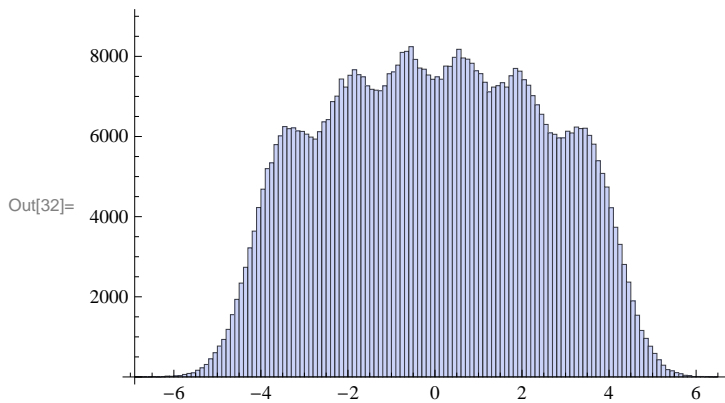
```
In[31]:= Plot[eigDensityGUE[6, x], {x, -6, 6}]
```

Out[31]=



Compared with a random sample...

```
In[32]:= Histogram[Flatten[Table[realeigs[randGUE[6]], {100 000}]]]
```

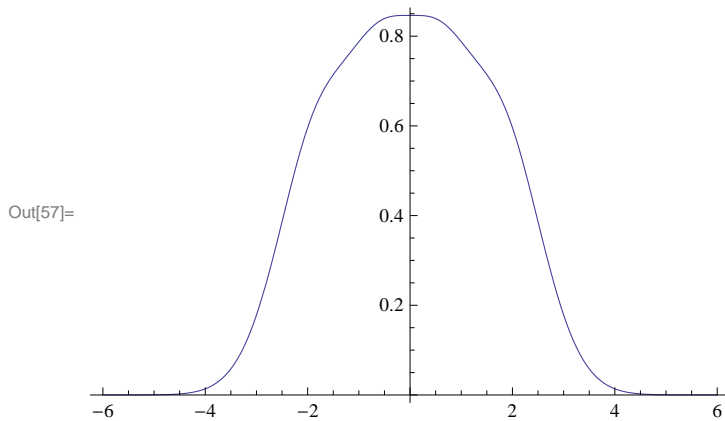


Now the eigenvalue density for the GOE (for even N) is  $\sum_{k=0}^{N/2-1} [\phi_{2k}(x)^2 - \phi'_{2k}(x) \int_0^x \phi_{2k}(t) dt]$ .

```
In[48]:= eigDensityGOE[N_, x_] := Sum[\phi[2 k, x]^2 - \phi'_{2 k}(x) \int_0^x \phi_{2 k}(t) dt, {k, 0, N / 2 - 1}];
```

The theoretical density for N=6...

```
In[59]:= interpEigDensityGOE[n_] :=
  Interpolation[Table[{x, eigDensityGOE[n, x]}, {x, -6.0, 6.0, 0.05}]];
interpEigDensityGOE6 = interpEigDensityGOE[6];
Plot[interpEigDensityGOE6[x], {x, -6, 6}]
```



Compared with a random sample...

```
In[58]:= Histogram[Flatten[Table[realeigs[randGOE[6]], {100 000}]]]
```

