Department of Mechanical Engineering
Massachusetts Institute of Technology

2.010 Modeling, Dynamics and Control III
Spring 2002

SOLUTIONS: Problem Set #1

Problem 1

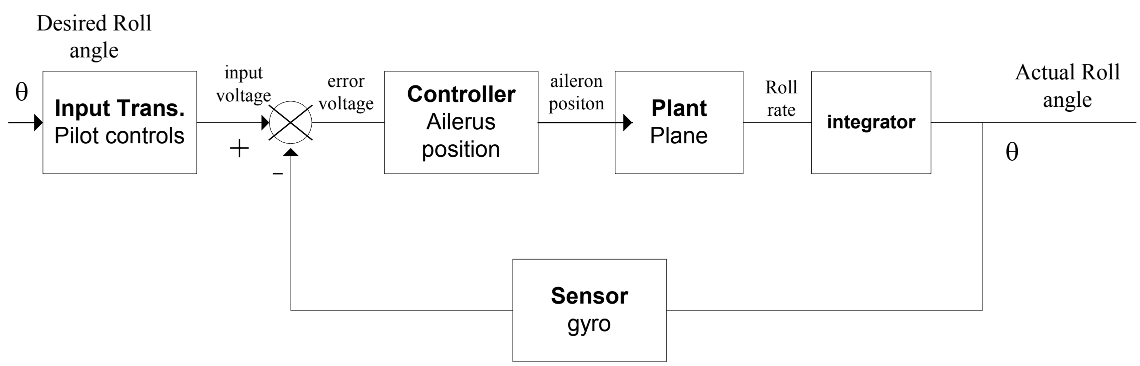
This problem is aimed at helping you picture the different ways in which controls fit into everyday life. This answers are not exhaustive of the possibilities. In particular for the robot welder, there are many other possibilities on what to control in addition to position, such as strength of weld, duration, speed, heat used.

		Water closet	**Refrigerator**	**Shower mixer**	**Robot Welder**	**Iris of your eye**
a	**Engineering/ customer goal**	To flush toilet each time with appropriate volume, refill tank	Keep food cold so it won't spoil	Mixing of hot and cold water to maintain desired temp. and volume	Make the weld in the correct position each time	To let in the appropriate amount of light to be able to see properly
b	**A measure of good performance**	Height of water in tank after each flush	Maintain temperature inside relatively constant	Proper amount of flow at a constant temp.	Position of weld from desired position	Being able to adjust to see well in bright light and in the dark.
c	**Output variable**	Water height	Temperature inside box	Volume rate and temperature of water	Position of weld	Size of opening (pupil)
d	**Reference input**	Water height set by floater	Thermostat	Position of shower knob. (this input is usually not in degrees, but rather relative position)	Position of where weld should be as programmed by the user	Image seen by brain
e	**Plant and actuator**	Water tank and valve	Icebox and cooling system	Shower head, knob, valve	Robot and piece to weld	Eye ball, optic nerve, iris muscles
f	**Measurement**	Floater	Thermometer	The user feels the correct water temp, volume rate	Some type of position sensor	Amount of light in the eye as sensed by the brain
g	**Disturbance**	Floater gets stuck Valve is broken Toilet is clogged	Opening the door Inserting warm objects	Loss of hot water in water input. Drop in water pressure	Vibration Position change of piece	Sudden burst of light, dilated pupils

Problem 2

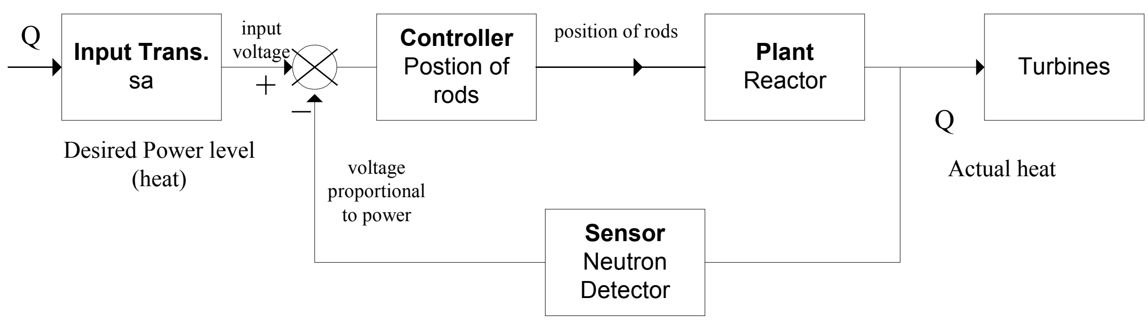
Functional Block Diagram:

Input Transducer: Pilot will input a desired angle through his controls which will be converted to an input voltage.
Output Transducer: Gyroscope measures the actual angle and converts it to a voltage
Controller Aileron position
Plant Plane dynamics



Problem 3

Functional Block Diagram



It is important to understand and appreciate the difference between functional block diagrams and mathematical block diagrams. The latter will be the focus of this course, but Functional block diagrams are particularly useful in the early design process of your control systems.

Problem 4

a) Time Constant

Note that this is a **first order** system which can be rewritten to fit the general form of:

$$\frac{d}{dt}X + \frac{1}{\tau}X = \text{input}$$

The equation becomes:

$$\frac{d}{dt}T + \frac{1}{60}T = \frac{1}{18}Q_{in}$$

Then it is obvious that $\tau = 60$.

b) Steady State Gain (K)

You can find the steady state gain by letting all the transients go to zero. Namely $\frac{d}{dt} = 0$.

Therefore:

$$K = \frac{T}{Q} = \frac{10}{3}$$

c) Transfer Function

Find the transfer function by applying the Laplace transform:

$$sT(s) + \frac{1}{60}T(s) = \frac{1}{18}Q(s)$$

Then solving for the transfer function

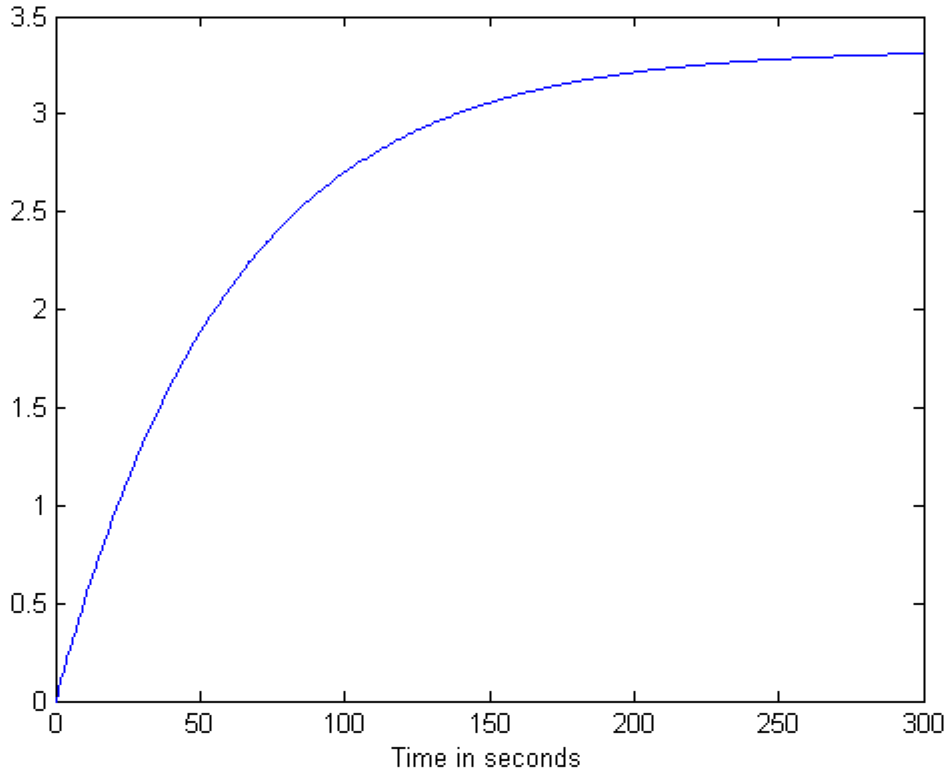
$$G(s) = \frac{T(s)}{Q(s)} = \frac{1/18}{s + 1/60}.$$

d) Plot

The equation for the curve is

$$T(t) = \frac{10}{3} - \frac{10}{3}e^{-\frac{t}{60}}$$

Value of t/τ	**t**	**Value of function**
1	60	2.0865
2	120	2.8746
3	180	3.1646
4	240	3.2713
5	300	3.3105


3.5
3
2.5
2
1.5
1
0.5
0
0
50
100
150
200
250
300
Time in seconds

Problem 5
Note that this is a **second order** system.
a) Equation of motion

Equation of motion is found by doing a force balance on the system

$$F_m = F_x + F_k$$
$$m\ddot{x}_2 = k(x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2)$$

which is the same as:

$$m\dot{v}_2 = k(\int v_1 - \int v_2) + b(v_1 - v_2)$$

b) Transfer function

The transfer function can be obtained by taking the Laplace transform

$$G(s) = \frac{Y(s)}{U(s)} = \frac{V_2(s)}{V_1(s)} = \frac{k + bs}{ms^2 + bs + k}$$

c) System Parameters
Given the system parameters: k = 400 N/m, m = 1 Kg, b = 300 N/m/sec
We can calculate all the relevant information about the system.
First we change the denominator so that it looks like the general second order form, namely:

$$s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}$$

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{k/m + b/m\, s}{s^2 + (b/m)s + (k/m)}$$

Then we can easily figure out what each term is:

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{300s + 400}{s^2 + 300s + 400}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400\ N/m}{1\ Kg}} = 20 \text{ Hz}$$

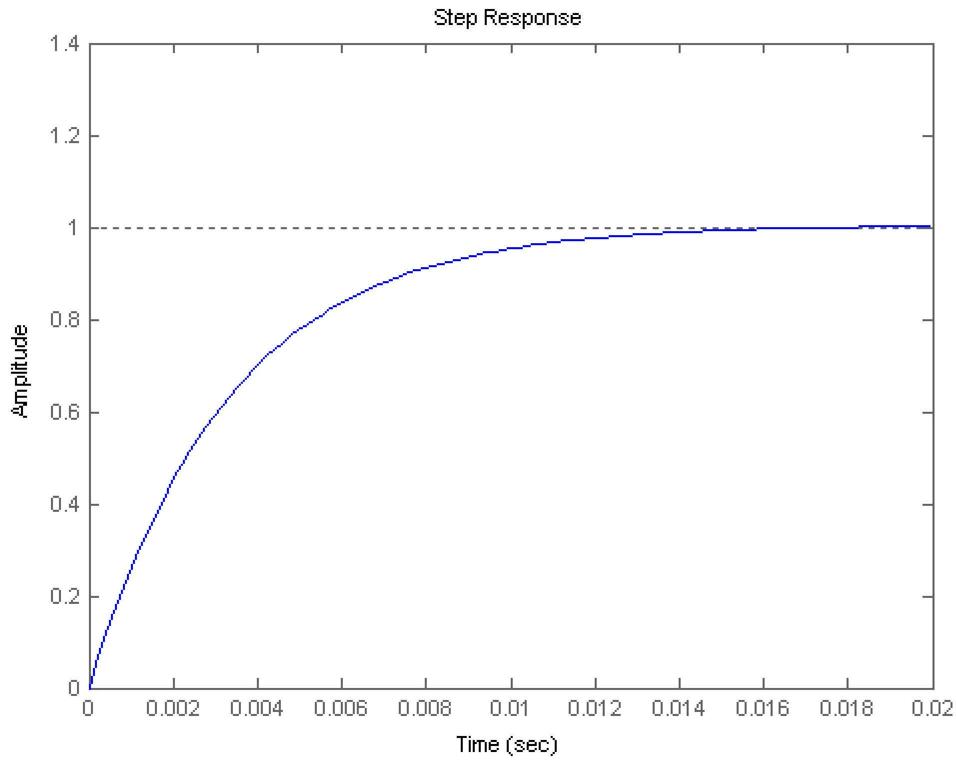
$$\zeta = \frac{1}{2\omega_n}\frac{b}{m} = \frac{1}{40\ Hz}\frac{300\ N/m/\sec}{1\ Kg} = 7.5 \text{ (VERY overdamped)}$$

Approximation for the 2% settling time for a second order system is given by:

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{7.5 \times 20} = 0.0267$$

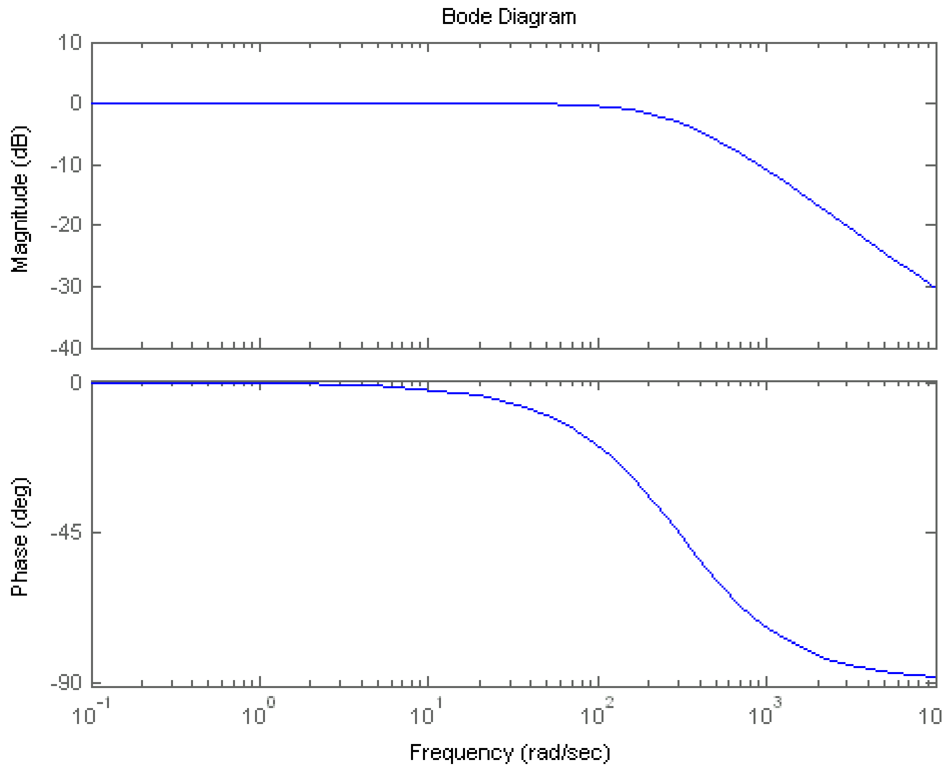
d) Step Response

$$T(t) = L^{-1}\left\{\frac{300s + 400}{s^2 + 300s + 400}\frac{1}{s}\right\}$$



f) Bode Plot

Notice that since the system is highly overdamped there is no peak for this second order system.



g) Resonant Frequency

The resonant frequency of a second order system is at the peak of the magnitude of the bode plot. Since the system is overdamped it will not resonate.