| READING |
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| Chapter 4<br>4.1 – 4.6, 4.8,<br>2.10, 2.11, 3.7 |

# Department of Mechanical Engineering
# Massachusetts Institute of Technology

**2.010 Modeling, Dynamics and Control III**
**Spring 2002**

Problem Set #3

**DUE** Thursday, February 28, 2002

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**Problem 1**
Nise Problem 4-26
For this problem plot the step response of $\theta_2$ by using MATLAB, and verify the results obtained in part b.

**Problem 2**
Nise Problem 4-29

**Problem 3**

Some large systems, such as supertankers and other vehicles when steered from the rear, can exhibit what is called: non-minimum phase behavior. If you are an inexperienced helmsman, this means the ship will seem to behave in a counter-intuitive fashion. We can see why this happens by looking at the transfer function.

A simple model of the rate at which the ship's heading, $\Omega$, in response to the rudder angle, $\theta_r$, is given by:

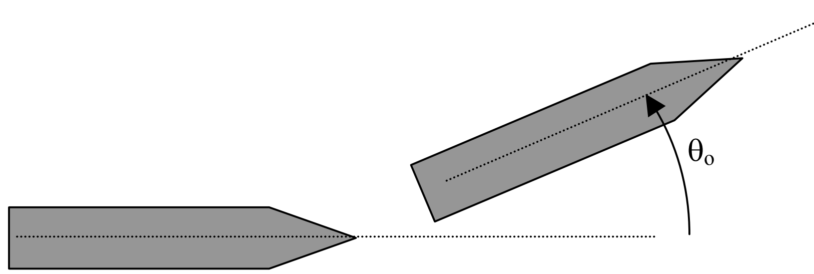
$$\frac{\Omega(s)}{\theta_r(s)} = \frac{K(-s+b)}{(s+a)}$$

Based on this simple model with a = 0.001, b = 1 and K = 0.001

a) How will the ship react to a step change in the rudder angle (complement your answer with a sketch of the time response)
b) What is the heading angle as a function of time?
c) Why is this response counter intuitive?
d) How long will it take this ship to reach a constant turning rate after the rudder is changed?

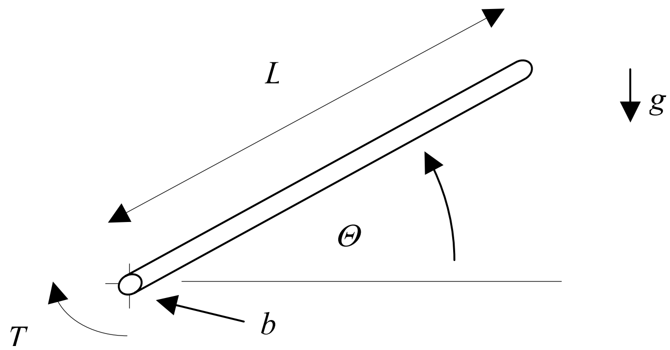
Because of the difficulty of hitting exactly the right heading with such a slow system, we want to develop a "heading controller" for the helmsman. This means the input would be a desired heading, $\theta_o$, instead of a turning rate.

e) Draw a block diagram for a proportional control system to control the ship heading angle, $\theta_o$, given the above transfer function.
f) Is there any value for the controller gain, K, that will give an acceptable response? Please explain



**Problem 4**

When controlling a simple 1-degree of freedom robot 'arm', the system can be modeled as a beam with a torque, T, applied at the hinged end. The torque imposes a rotation denoted by the angle, $\theta$. Assume damping, b, at the hinged end *cannot* be neglected.




If the beam is of uniform cross section and has a mass, m, and length, L:

a) Derive the full *non-linear* model for this 'arm' assuming a perfect torque source
b) Derive three <u>linear</u> transfer functions for the three different nominal conditions:
    1. The arm pointing vertically down
    2. The arm pointing vertically up
    3. The arm horizontal
c) Compare these three expressions and comment on how the *small motion* dynamic behavior will differ.
d) Apply simple proportional control to each system (close the loop with a controller = $K_c$ ) and find a value of the controller gain, $K_c$, that will yield the same natural frequency for all three closed-loop systems.
e) How are these gains different and why?
f) How would you design a control sytsem to operate the same at all nominal angles?