READING

Chapter 4 and Chapter 5

Department of Mechanical Engineering
Massachusetts Institute of Technology
2.010 Modeling, Dynamics and Control III
Spring 2002

Problem Set #4

DUE Thursday, March 7, 2002

Problem 1
Nise Problem 5-3

Problem 2
Nise Problem 5-48

Problem 3
Nise Problem 4-20: Parts a. and b. only.

Problem 4
Nise Problem 4-23: Parts b. and c. only.

Problem 5
Consider the following second-order system with an extra pole:

$$H(s) = \frac{\omega_n^2 p}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$

Show that the unit step response is

$$y(t) = 1 + Ae^{-pt} + Be^{-\sigma t}\sin(\omega_d t - \theta)$$

where

$$A = \frac{-\omega_n^2}{\omega_n^2 - 2\zeta\omega_n p + p^2}$$

$$B = \frac{p}{\sqrt{(p^2 - 2\zeta\omega_n p + \omega_n^2)(1-\zeta^2)}}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{p - \zeta\omega_n}$$

a) Which term dominates *y(t)* as *p* gets large?
b) Give approximate values for *A* and *B* for small values of *p*
c) Which term dominates as *p* gets small? (Small with respect to what?)
d) Using the explicit expression for y(t) above or the `step` command in MATLAB, and assuming $\omega_n = 1$ and $\zeta = 0.7$, plot the step response of the system above for several values of p ranging from very small to very large. At what point does the extra pole cease to have much effect on the system response?