Massachusetts Institute of Technology
Department of Mechanical Engineering
2.010 Modeling, Dynamics, and Control III
Spring 2002
Problem Set #5
Due Thursday, March 14, 2002

Reading
Nise Chapter
6.1~6.4

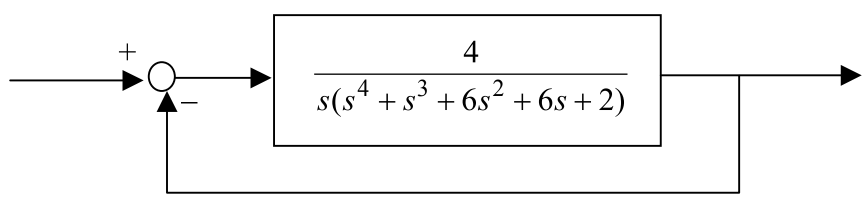
Problem 1

Using the Routh-Hurwitz stability criterion, examine the stability of the following open-loop system and determine how many poles are in the right half plane.

$$G(s) = \frac{2s^2 + 3s + 8}{s^4 + 2s^3 + 4s^2 + 6s + 8}$$

Problem 2

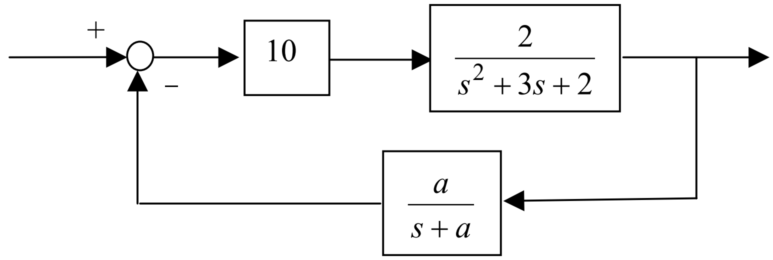
Use the Routh-Hurwitz stability criterion to determine whether the unity feedback system shown below is stable. If unstable, how many closed loop poles are in the right half plane?



Problem 3 Nise Problem 6-38

Problem 4

Shown below is a control system with a noise filter in its feedback loop. When a sensor signal is noisy, we often use a low-pass filter to smooth out the signal. Care must be taken, however, since such a filter may cause instability. Using the Routh-Hurwitz stability criterion, determine the range of the filter parameter *a* to keep the closed loop system stable.



Problem 5

A remote manipulator is a robotic device to perform a task in the distance. The three schematic diagrams shown below illustrate different configurations of one degree-of-freedom remote manipulators controlled by a human operator. The one in Figure 1 is a primitive architecture where the robotic arm replicates the motion of the human operating a joystick. The displacement sensor attached to the joystick can measure the human hand motion, and the measured signal is sent to the remote manipulator for reproducing the human motion.

In this simple system the human operator has no means to observe interactions between the robotic arm and the environment. The human does not know, for example, whether the robotic arm is in contact with the environment. To better observe such physical contacts, force-reflective control has been developed for remote manipulators. As shown in Figure 2, the contact force at the tip of the robotic arm is measured with a force sensor, and the measured force is sent back to the human operator side. To present the measured force to the human operator, the joystick is equipped with an actuator, generating a torque equivalent to the measured contact force. Therefore, when the robotic arm is in contact with the environment, the actuator on the joystick tends to push back the human hand and thereby the human can detect contact with the environment surface. This force-reflective control, however, often leads to instability. The following is to analyze the system's stability.

Let J_r and θ_r be the inertia and the angular displacement of the robotic arm, and J_j and θ_j be the ones of the joystick arm, respectively. The robotic arm is controlled in such a way that the discrepancy between θ_r and θ_j be reduced; the torque generated by the robot actuator is proportional to the discrepancy and is given by $\tau_r = k_r(\theta_j - \theta_r)$. The joystick side actuator, on the other hand, generates a torque proportional to the negative of contact force F, i.e. $\tau_j = -k_f F$. The environment surface is modeled as a pure capacitive element with stiffness K_e. For the sake of simplicity, assume no energy dissipative element involved in the system. Answer the following questions.

> **Question a).** Obtain equations of motion for both the robotic arm and the joystick system, and show that the transfer function from the torque applied by the human operator, τ_h, to the angular displacement of the joystick, θ_j, is given by

$$G_1(s) = \frac{\theta_j(s)}{\tau_h(s)} = \frac{J_r s^2 + k_r + K_e}{J_r J_j s^4 + J_j(k_r + K_e)s^2 + k_f k_r K_e} \quad (1)$$

In the following questions, use the dimension-less parameter values given by:

$$J_r = 2, J_j = 1, k_r = 2, K_e = 8, k_f = 0.5. \quad (2)$$

The human operator conceives a desired displacement θ_h and generates torque τ_h in such a way that the discrepancy between θ_h and θ_j be reduced. Let us assume that this human operation is modeled as a type of Proportional-and-Derivative control given by

$$\tau_h(s) = K(1+k_v s)[\theta_h(s) - \theta_j(s)] \tag{3}$$

> **Question b).** Obtain the closed-loop transfer function of the overall system including the PD control by the human operator and the above robot-joystick system given by Eqs.(1) and (2). Using the Routh-Hurwitz stability criterion, show that the closed-loop system is unstable for all PD gains, $K \neq 0, k_v \neq 0$.

To overcome the instability problem, a different type of architecture, called *bilateral servo,* has been proposed. See Figure 3. Instead of feeding back the contact force, bilateral servo uses a different signal for rendering the physical contact between the robot and the environment. When the human wants to push the robot arm forward but the robot gets stuck due to contact with the environment, the discrepancy between θ_r and θ_j tends to increase. Therefore, this discrepancy signal can be used for indicating contact with the environment. As shown in the figure, the difference $\theta_j - \theta_r$ is fed back not only to the robot actuator but also to the joystick actuator with the opposite sign. Namely, the joystick actuator generates a torque in proportion to the difference: $\tau_j = -k_j(\theta_j - \theta_r)$, where k_j is a gain set to: $k_j = 2$.

> **Question c).** Show that in the bilateral servo control system the transfer function from the torque applied by the human operator, τ_h, to the angular displacement of the joystick, θ_j, is given by
>
> $$G_2(s) = \frac{\theta_j(s)}{\tau_h(s)} = \frac{J_r s^2 + k_r + K_e}{J_r J_j s^4 + [J_r k_j + J_j(k_r + K_e)]s^2 + k_j K_e} \tag{4}$$
>
> Assume that the human control behavior is the same as Eq.(3), i.e. PD control. Using the parameter values given in Eq.(2) and setting $k_j = 2$, obtain the closed-loop transfer function. Using the Routh-Hurwitz stability criterion, show that the new control system is stable for all positive PD gains, $K > 0, k_v > 0$.

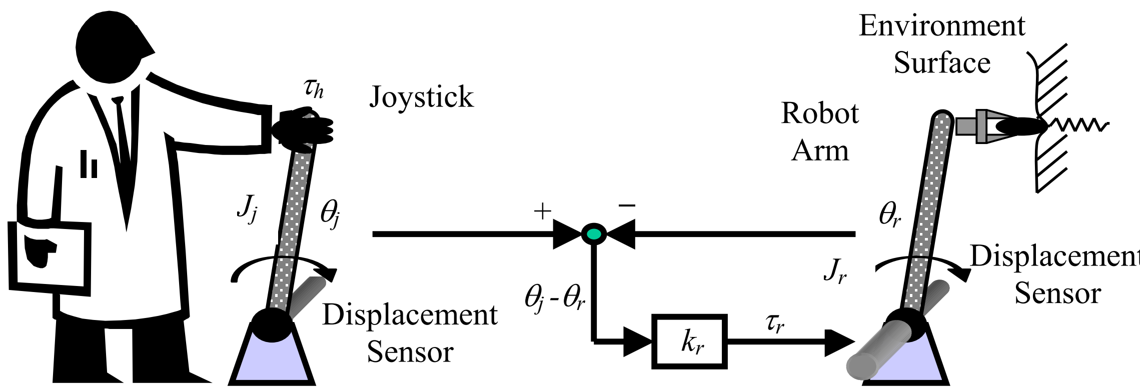


Figure 1 Remote manipulator with no force feedback

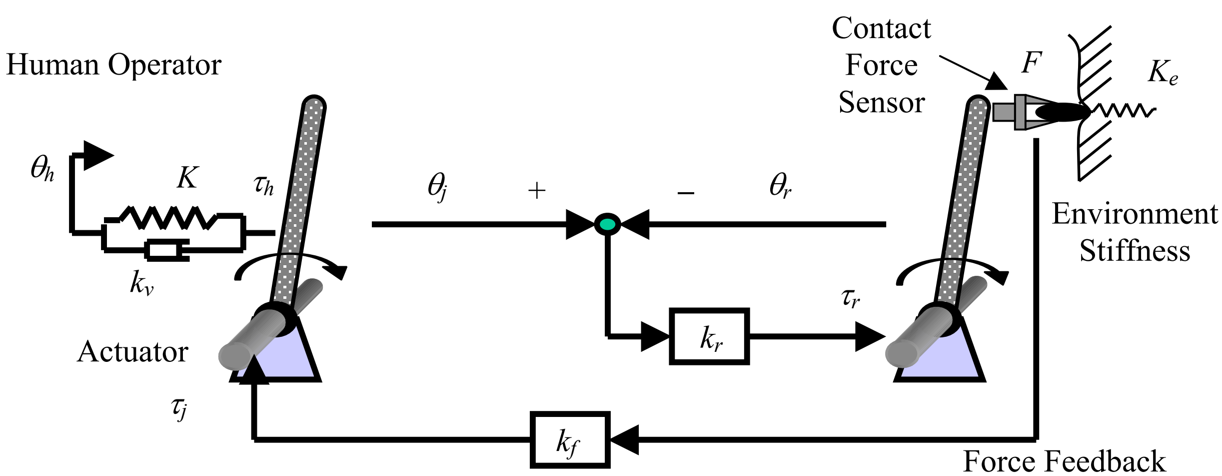


Figure 2 Force-reflective control of remote manipulator

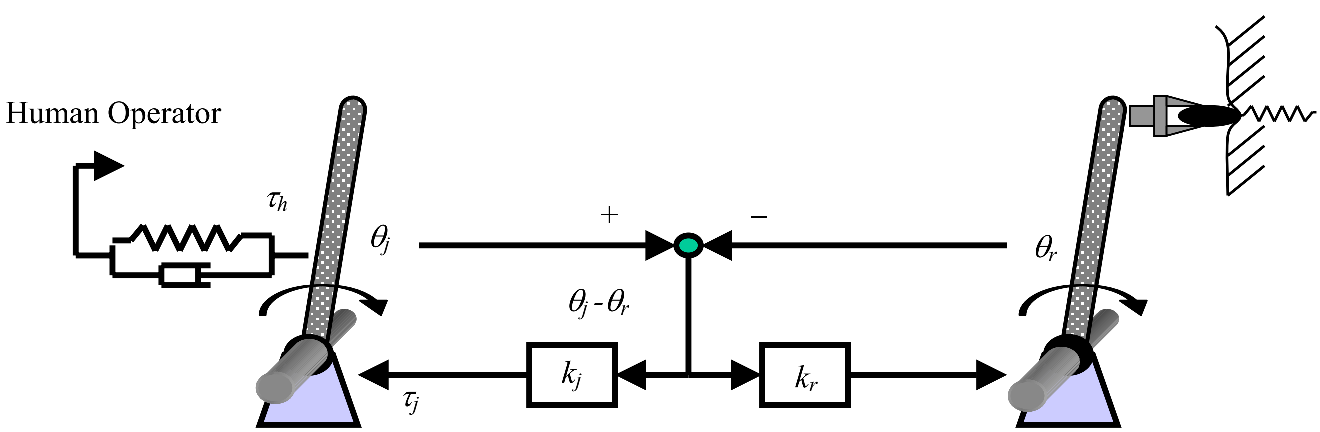


Figure 3 Bilateral servo control of remote manipulator