Department of Mechanical Engineering
Massachusetts Institute of Technology
2.010 Modeling, Dynamics and Control III
Spring 2002

SOLUTIONS: Problem Set # 6

Problem 1
Nise Problem 7-1

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

Steady state errors for the following inputs: $25u(t)$, $37tu(t)$, $47t^2u(t)$.

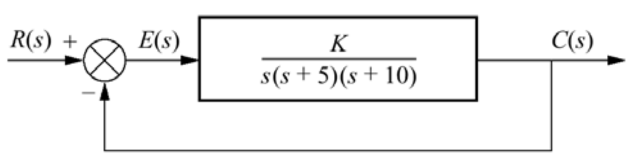
$$e(\infty) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

Input	R(s)	Steady state error $e(\infty)$
$25u(t)$	25/s	0
$37tu(t)$	$37/s^2$	6.075×10^{-2}
$47t^2u(t)$	$94/s^3$	∞

For the ramp input the calculation is:

$$\lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{37s[s(s+38)(s^2+2s+28)]}{s^2[s(s+38)(s^2+2s+28)+450(s+8)(s+12)(s+15)]} = \frac{(37)(38)(28)}{(450)(8)(12)(15)} = 0.06075$$

Problem 2
Nise Problem 7-16



a) Find the value of K to yield a steady-state position error of 0.01for the input (1/10)t.

$$e(\infty) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

$$R(s) = \frac{1/10}{s^2}$$

$$G(s) = \frac{K}{s(s+5)(s+10)}$$

$$\frac{sR(s)}{1+G(s)} = \frac{1/10(s+5)(s+10)}{s(s+5)(s+10)+K}$$

Taking the limit we get:

$$\lim_{s\to 0} s\,E(s) = \frac{5}{K}$$

For the steady state error to be 0.01, we need

$$\boxed{K = 500}$$

b)

$$K_v = \lim_{s\to 0} sG(s) = \frac{K}{(s+5)(s+10)} = \frac{500}{50}$$

$$\boxed{K_v = 10}$$

c)

$$K_v = \frac{K}{50}$$

$$e(\infty) = \frac{1/10}{K_v} = \frac{5}{K}$$

The smallest steady state error will be given by the largest stable K. To find this K use the Routh Hurwitz table

Characteristic equation :

$$s(s+5)(s+10) + K = s^3 + 15s^2 + 50s + K$$

$$C.E. = s^3 + 15s^2 + 50s + K$$

s^3	1	50
s^2	15	K
s^1	50 - K/15	0
s^0	K	0

System will be marginally stable when K = 750

Therefore:

$$\boxed{e(\infty) = \frac{5}{750} = 0.0067}$$

Problem 3
Nise Problem 7-28

$$\text{Given } \; G(s) = \frac{K}{s^n (s+a)}$$

Find values for n, K, and a that yield a 10% O.S. and $K_v = 100$.

In order to have K_v be a constant we need to have a system of Type = 1.

$$\therefore n = 1$$

$$G(s) = \frac{K}{s(s+a)}$$

The closed loop transfer function will be given by:

$$T(s) = \frac{K}{s^2 + as + K}$$

Velocity constant

$$K_v = \lim_{s \to 0} sG(s) = \frac{K}{a} = 100$$

For a 10% O.S we have

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.6$$

Now we know that:

$$a = 2\zeta\omega_n \text{ and } \omega_n^2 = K$$

Therefore:

$$a = 1.2\sqrt{K}$$

Now we have two equations with two unknowns. So we can solve for a and K.

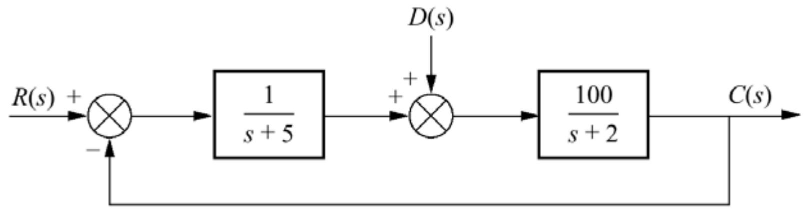
$$a = 144$$

$$K = 14400$$

$$\boxed{G(s) = \frac{14400}{s(s+144)}}$$

Problem 4
Nise Problem 7-34

Find the total steady state error due to the unit step input and the unit step disturbance



$$G_1(s) = \frac{1}{s+5} \qquad G_2(s) = \frac{100}{s+2}$$

The steady state error for disturbances is given by:

$$e(\infty) = \lim_{s \to 0} \frac{sR(s) - sD(s)G_2(s)}{1 + G_1(s)G_2(s)}$$

Since we are dealing with unit step input and disturbance

$$R(s) = D(s) = \frac{1}{s}$$

So the steady state error is:

$$e(\infty) = \lim_{s \to 0} \frac{1 - \frac{100}{s+2}}{1 + \frac{1}{s+5}\frac{100}{s+2}} = -\frac{49}{11} = -4.4545$$

$$\boxed{e(\infty) = -4.4545}$$