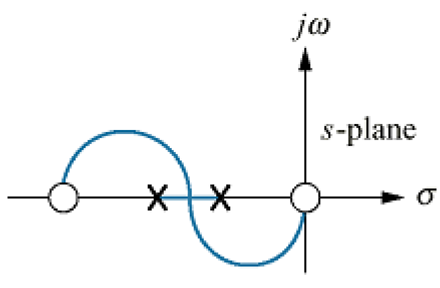
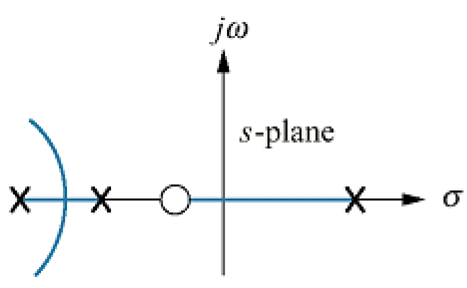
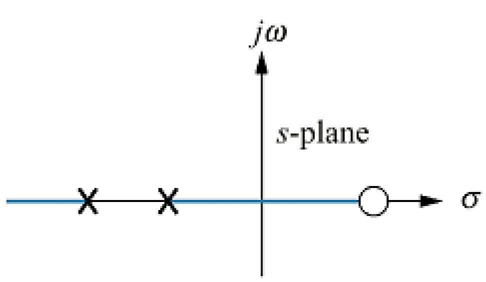
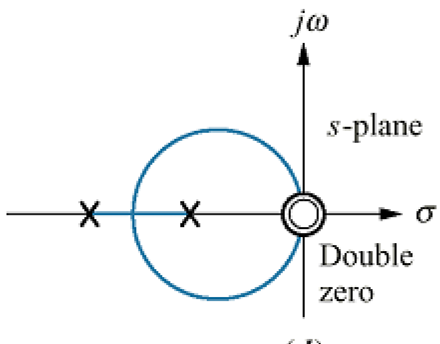
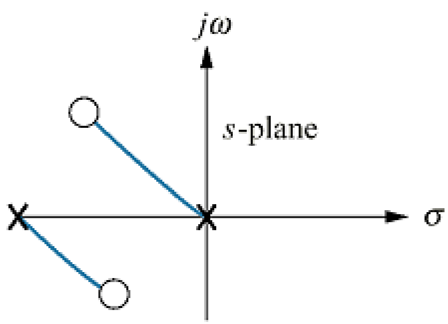
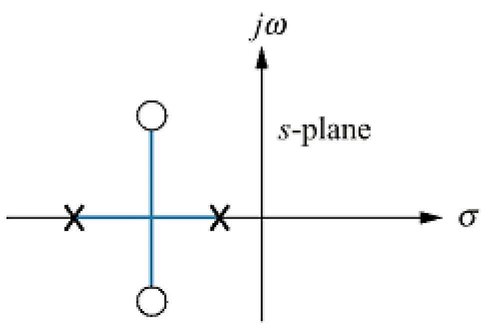
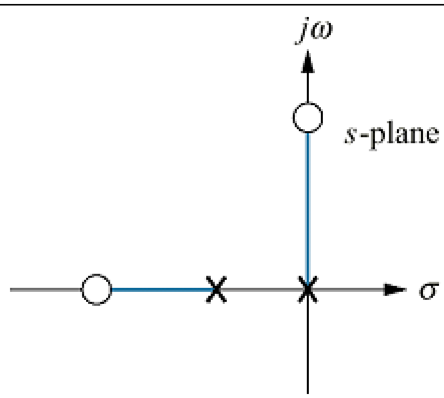
Department of Mechanical Engineering
Massachusetts Institute of Technology
2.010 Modeling, Dynamics and Control III
Spring 2002

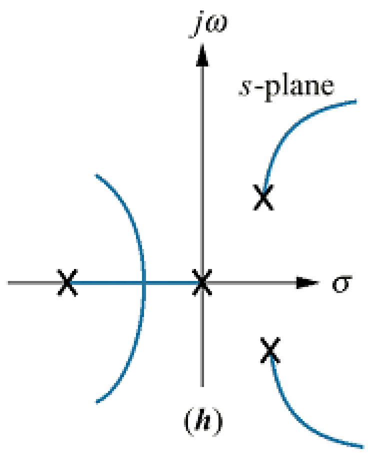
SOLUTIONS: Problem Set # 7

Problem 1 Nise Problem 8-1

Say whether the picture can be root locus or not

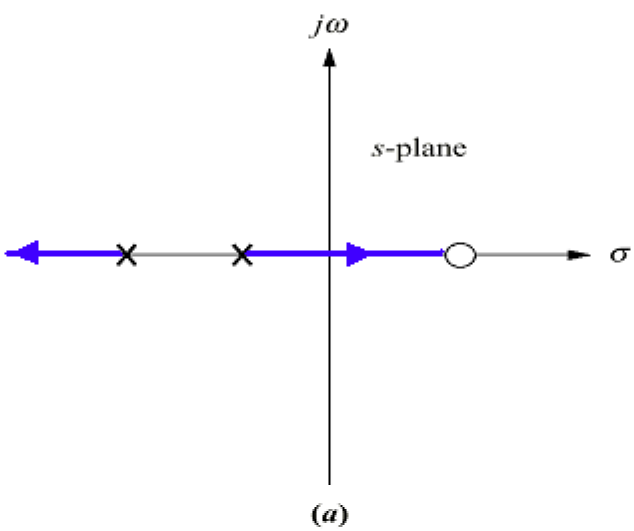
 (*a*)	No. Not symmetrical Real axis to left of an even number of poles and zeros
 (*b*)	Yes.
 (*c*)	Yes.

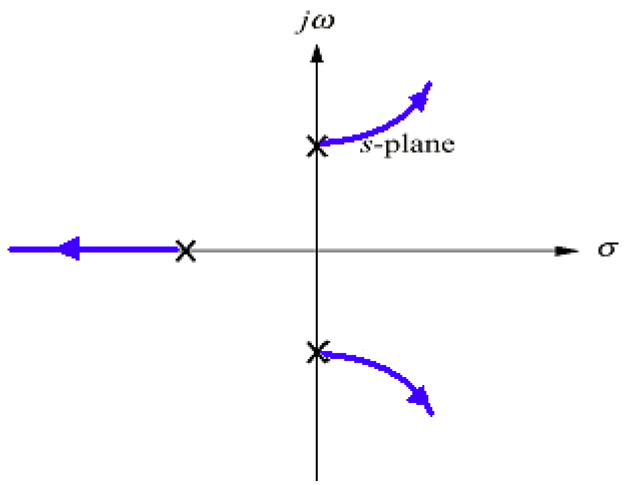
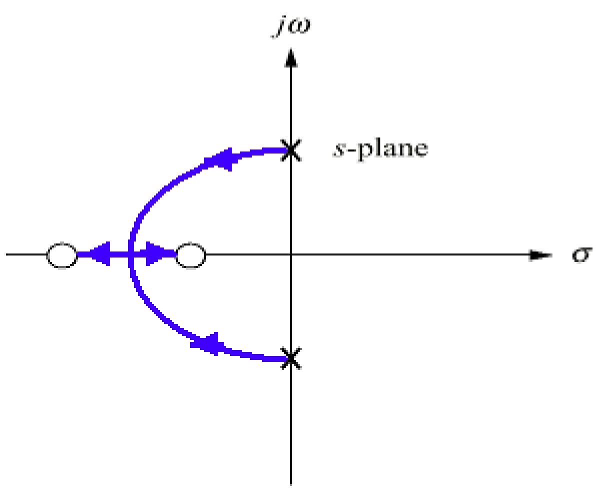
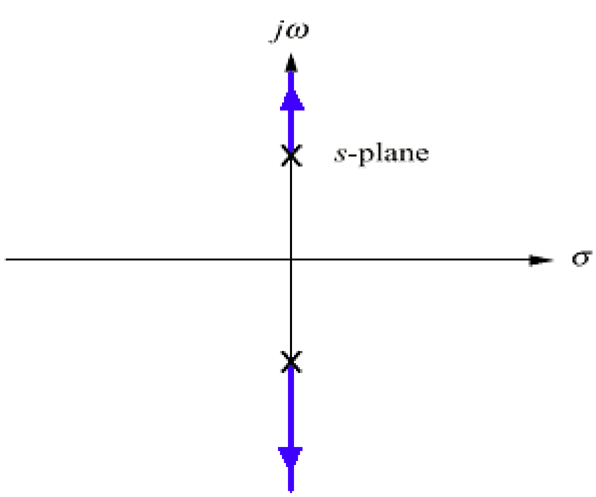
 (d)	Yes.
 (e)	No. Not symmetric. Missing real axis for odd number of poles to the left Poles not complex conjugates.
 (f)	Yes.
 (g)	No. Not symmetric On real axis missing segment to the left of an odd number of poles

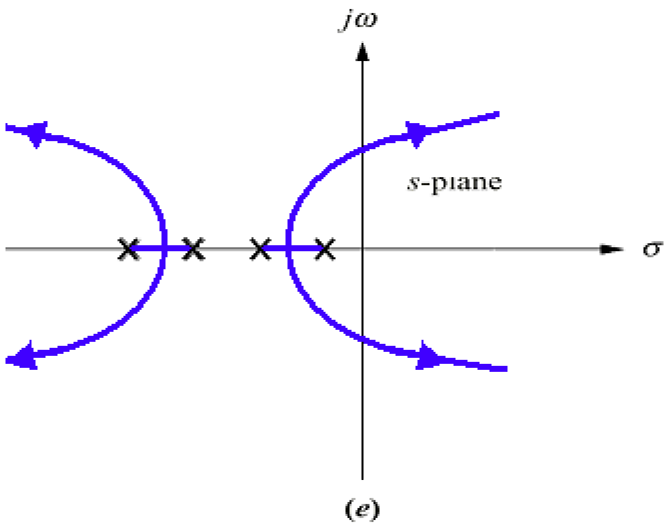
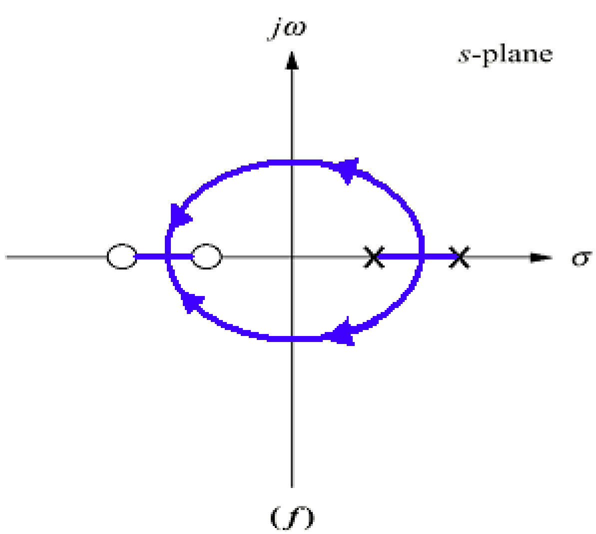
	Yes.

Problem 2 Nise Problem 8-2

Plot the root locus

	Start by connecting the segments on the real axis. In this case once you do that, you are done!

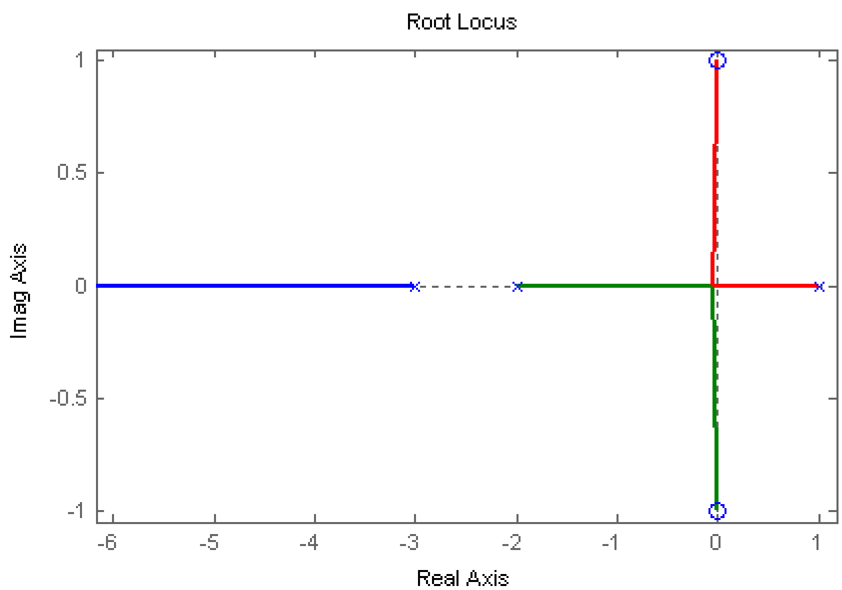
 (b)	**Step 1:** Connect the segment on the real axis. (Only one towards infinity) **Step 2:** Determine how many asymptotes R-L will have. (#poles - #zeros = 3) **Step 3:** Draw the asymptotes. (start about a third of the way towards the first pole) **Step 4:** Draw the Root Locus coming out of the poles going towards infinity along the asymptotes
 (c)	**Step 1:** Connect the segment on the real axis. **Step 2:** Determine how many asymptotes R-L will have. (2 – 2 = 0) **Step 3:** Each branch will end at a zero since there are no asympotes. **Step 4:** Draw the Root Locus coming out of the poles going towards infinity along the asymptotes
 (d)	**Step 1:** Connect the segment on the real axis. There isn't any! **Step 2:** Determine how many asymptotes R-L will have. (2 – 0 = 2) **Step 3:** The asymptotes will orginate at 0 and go towards infinity. **Step 4:** Draw the Root Locus along the asymptotes.

 (e)	**Step 1:** Connect the segment on the real axis. **Step 2:** Determine how many asymptotes R-L will have. (4 – 0 = 4) **Step 3:** Asymptotes will originate between the two middle poles and will extend towards infinity at the angles: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **Step 4:** Draw the Root Locus coming out of the poles going towards infinity along the asymptotes
 (f)	**Step 1:** Connect the segment on the real axis. **Step 2:** Determine how many asymptotes R-L will have. (2 – 2 = 0) **Step 3:** Each branch will end at a zero since there are no asympotes. **Step 4:** Draw the Root Locus coming out of the poles going towards the zeros.

Problem 3 Nise Problem 8-12

Plot root locus, state for what values of K system is stable

$$G(s) = \frac{K(s^2+1)}{(s-1)(s+2)(s+3)}$$



To find the value of K were the system is stable we use the Routh Hurwitz table.

The closed loop transfer function is given by

$$T(s) = \frac{K(s^2+1)}{(s-1)(s+2)(s+3)+K(s^2+1)} = \frac{K(s^2+1)}{s^3+(4+K)s^2+s+(K-6)}$$

The Routh Hurwitz table is given by

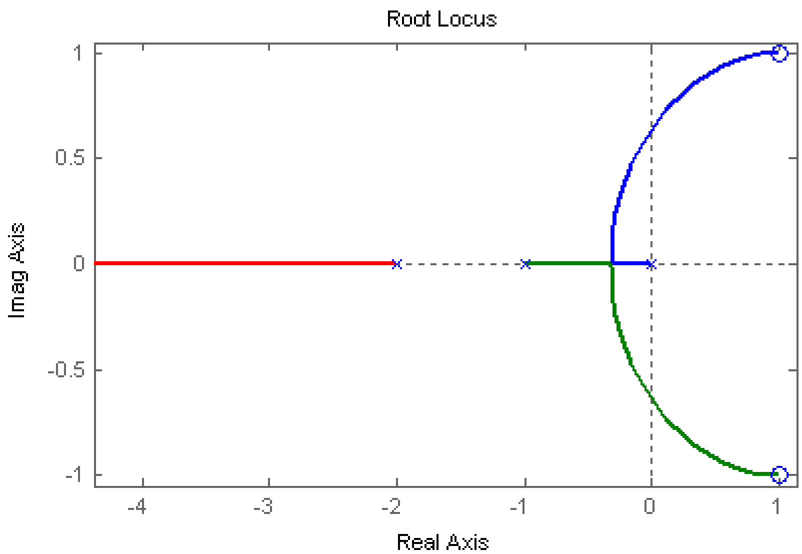
s^3	1	1
s^2	4+K	K-6
s^1	$\frac{10}{4+K}$	0
s^0	K-6	0

Now once we have plotted the Root Locus we see, that initially we will have an unstable system, since the pole is at –1 and then as K increases it will move over to the left hand side. By looking at the Routh table we see that for s^3, s^2 and s^1, will be positive for all values of K > 0. Now looking at the last line, we see that for K > 6 we will have a stable system. Also at the value of K = 6, the system will be marginally stable.

$$\boxed{K > 6}$$

b)

$$G(s) = \frac{K(s^2 - 2s + 2)}{s(s+1)(s+2)}$$



Look at the root locus unlike the previous problem here the poles start stable and *then* move towards instability when the poles move onto the right hand plane at a certain gain.

To find this gain, construct the routh hurwitz table again.

The characteristic equation is given by

$$C.E. = s^3 + (3+K)s^2 + (2-2K)s + 2K$$

s^3	1	2 - 2K
s^2	3+K	2K
s^1	$\frac{-2(K^2+3K-3)}{3+K}$	0
s^0	2K	0

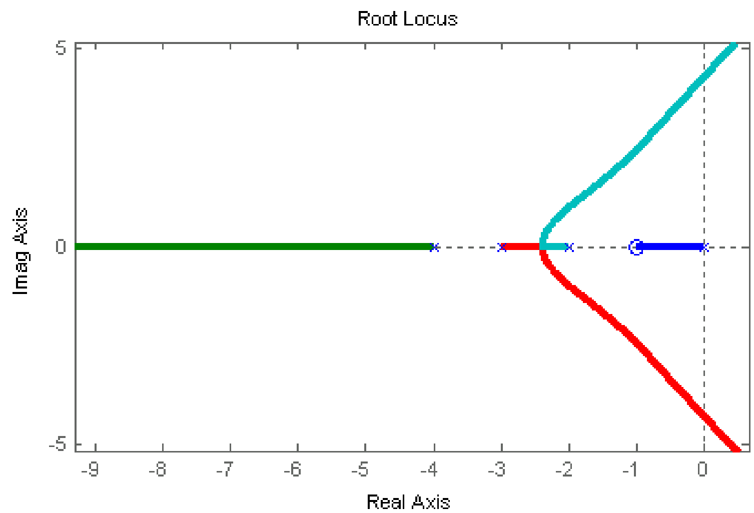
For stability we solve the quadratic $K^2 + 3K - 3 = 0$, and we see that for gains $K = 0.79, -3.79$, the system is marginally stable. Since we are only dealing with $K > 0$, the system will be stable for gains of

$$\boxed{0 < K < 0.79}$$

Problem 4 Nise Problem 8-17

$$G(s) = \frac{K(s+1)}{s(s+2)(s+3)(s+4)}$$

a) Root locus



b) Asymptotes

The asympotes can be found by weighing the position of the poles and zeroes

$$\alpha = \frac{z_1 - p_1 - p_2 - p_3 - p_4}{n-m} = \frac{1-2-3-4}{3} = \frac{8}{3}$$

So, the asymptotes are centered around the point 8/3 and since there are 3 they will come out at the angles 60°, -60° and 180°

c) K to make it marginally stable

$$C.E. = s^4 + 9s^3 + 26s^2 + (24+K)s + K$$

s^4	1	26	K
s^3	9	24+K	0
s^2	$\frac{K-210}{9}$	K	0
s^1	$\frac{K^2-105K-5040}{K-210}$	0	0
s^0	K	0	0

For the response to be marginally stable line s^1 must be zero. The values that make this row zero are:

$$K = 140.80, -35.80$$

Since K must be greater than zero, we know that $K = 140.80$ will make this equation line zero.

d)

Value of K to have a pole on the real axis at s = -0.5

$$K = -\frac{1}{G(s)} = \frac{s\cdot|s+2|\cdot|s+3|\cdot|s+4|}{|s+1|} = \frac{0.5(1.5)(2.5)(3.5)}{0.5} = 13.125$$