Massachusetts Institute of Technology
Department of Mechanical Engineering
2.010 Modeling, Dynamics, and Control III

Quiz #1

March 19, 2002
3:00 pm – 4:30 pm

Close book. Two sheets of notes are allowed.
Show how you arrived at your answer.

Problem 1

Consider a system with two feedback loops, as shown by the block diagram below.

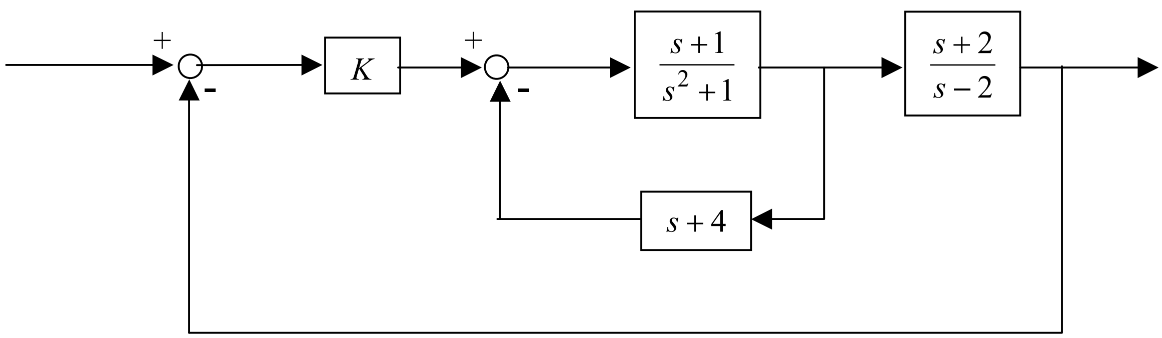

Figure 1 Original block diagram

(1-a). Reduce this block diagram to the standard unity feedback system shown in Figure 2, and obtain the transfer function $G(s)$.

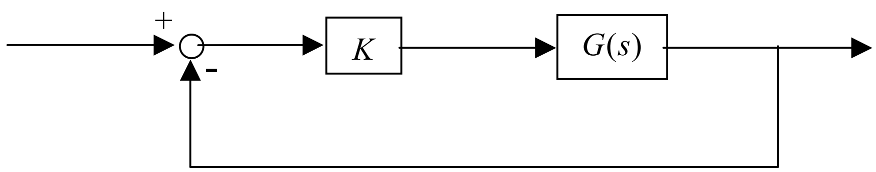

Figure 2 Reduced block diagram

(1-b). Obtain the open-loop poles and zeros of the system shown in Figure 2, and plot them on a complex plane. Is the open-loop system stable. Explain why.

(1-c). Using the Routh-Hurwitz stability criterion, obtain the range of feedback gain K to ensure the stability of the closed-loop system in Figure 2.

Problem 2

Space shuttles carry a remote manipulator system for various space missions. The figure below shows the schematic of a simplified one degree-of-freedom manipulator arm. The arm is driven by an actuator of rotor inertia J_1 through mass-less spur gears of gear ratio $1:N$, (i.e. the radius of the actuator-side gear is 1, while that of the arm side is N). The arm rotates about its shoulder joint having a torsional stiffness of K_t. The length of the arm is L and its inertia about the joint axis is J_2. The bearings holding the joint axis have a viscous damping of b, i.e., a drag moment proportional to the angular velocity, $-b\dot{\theta}_2$, acts on the joint axis. The arm is equipped with an optical range sensor measuring the arm tip position, y, relative to a fixture. A proportional feedback loop has been formed from the end point sensor, as shown in Figure 4. The transfer function $G(s)$ relates the arm tip position y to actuator torque τ. The proportional feedback gain is denoted k_p. Answer the following questions.

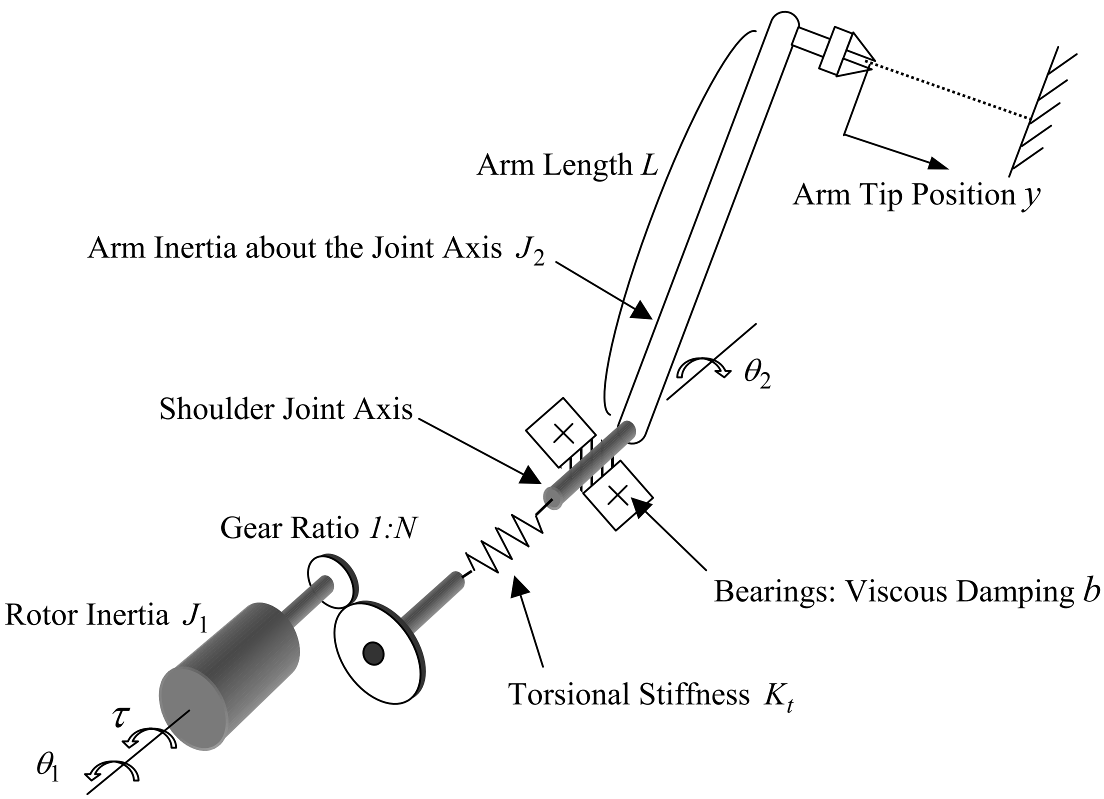

Figure 3 The schematic of one degree-of-freedom space shuttle manipulator

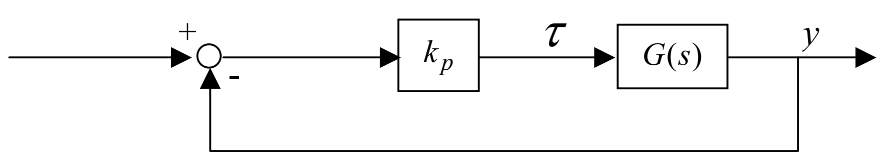

Figure 4 Proportional feedback of end point displacement y

(2-a). Obtain the equations of motion and show that the transfer function from actuator torque τ to the end point position y is given by the following equation:

$$G(s) = \frac{y(s)}{\tau(s)} = \frac{K_t L N}{s[N^2 J_1 J_2 s^3 + N^2 J_1 b s^2 + K_t (N^2 J_1 + J_2) s + K_t b]}$$

For the following two questions, use the transfer function with non-dimensional numerical values given by

$$G(s) = \frac{5}{s(s^3 + 12s^2 + 66s + 132)}$$

(2-b). As the proportional feedback gain k_p increased, the response of the system became oscillatory, and finally reached a marginally stable response. Find the gain k_p for which the system became marginally stable and obtain its oscillatory frequency.

(2-c). For gain $k_p = 41$, the closed-loop poles are:

$$p_{1,2} = -1 \pm j2, \quad p_{3,4} = -5 \pm j5$$

Which pole(s) dominate a step response? What is the approximate settling time of the system?