Massachusetts Institute of Technology
Department of Mechanical Engineering
2.010 Modeling, Dynamics, and Control III

Quiz #2

April 18, 2002
3:00 pm – 4:30 pm

Close book. Two sheets of notes are allowed.
Show how you arrived at your answer.
Show clearly which one is your final answer.
Calculators are allowed, but are not needed for the following questions.

Problem 1 (40%)

Consider the following third-order system with a unity feedback.

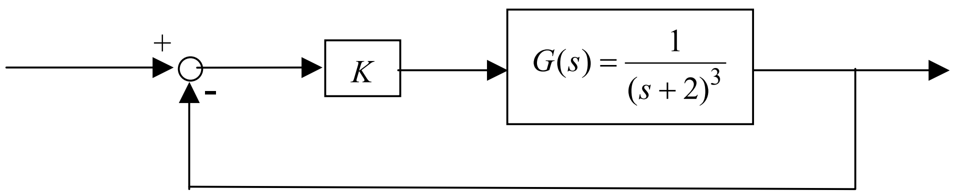

Figure 1 Proportional control of a third-order system

(1-a). Plot the open loop poles on a complex plane, and draw a root locus. (You can draw the exact root locus for this system without use of a computer.)

(1-b). Using the root locus plot, graphically obtain the feedback gain *K* that makes the closed loop system marginally stable. Verify the result by using the Routh-Hurwitz stability criterion.

(1-c). Find dominant closed-loop poles on the root locus that yield a peak time of $T_p = \frac{2\sqrt{3}\pi}{9}$. Obtain the feedback gain, *K,* satisfying the peak time specification. Also find the third closed pole on the real axis, and discuss whether the dominant second-order approximation is valid.

Problem 2 (60 %)

Shown below is a fire engine equipped with a powered ladder and a water pump system. When dashing water through the nozzle, a reaction force F_d works on the nozzle at the endpoint of the ladder, where the nozzle is secured. This reaction force acts as a disturbance to the ladder position control system regulating the ladder endpoint position, y. The length of the ladder is L, and its inertia about the joint axis is J. A hydraulic actuator generates torque τ on the joint axis. The bearings supporting the joint axis have a viscous damping of b. Assume that the ladder is a rigid body and that it is almost at the upright position; the joint angle θ varies within a small range near 90 degrees. Answer the following questions.

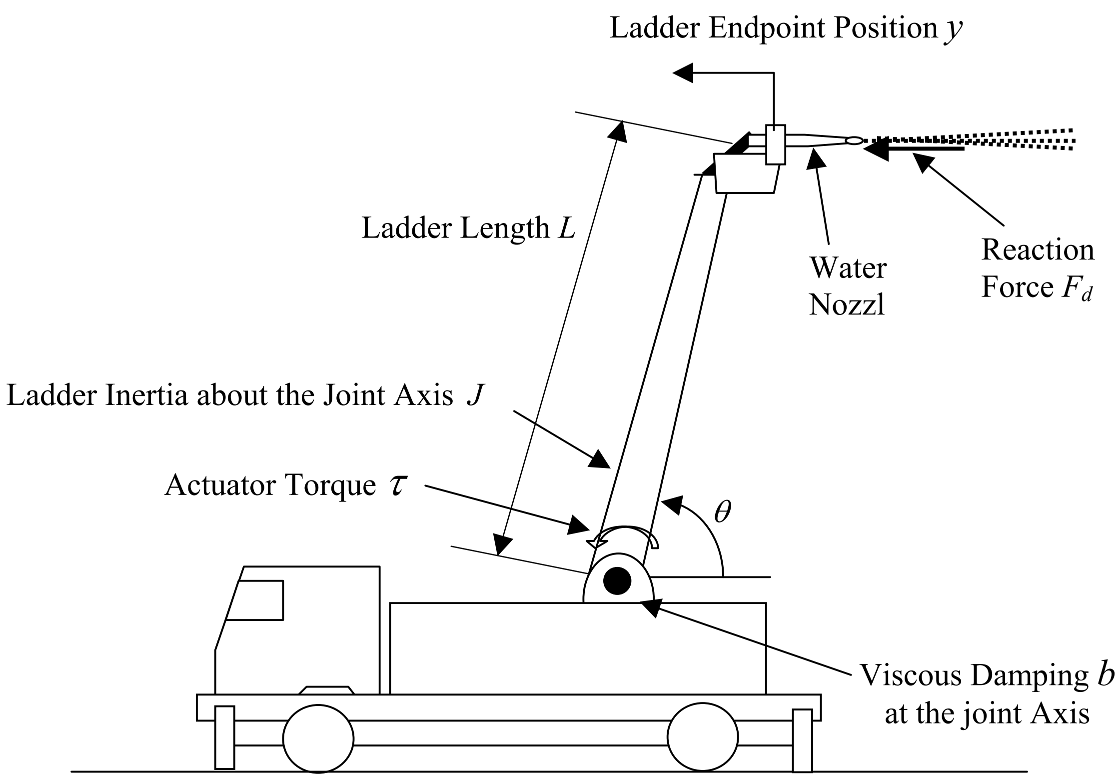

Figure 2 The schematic of a fire engine

(2-a). Obtain the equation of motion and show that, in Laplace form, the ladder endpoint position y is given by the following transfer function of actuator torque τ and nozzle reaction force F_d:

$$y(s) = \frac{L\tau(s) + L^2 F_d(s)}{s(Js + b)}$$

Use the following transfer function for the rest of the questions:

$$y(s) = \frac{\tau + 2F_d}{10s(s+4)}$$

(2-b). Consider a Proportional-plus-Derivative controller, as shown in Figure 3. We want to tune this control system so that the settling time will be $T_s = 1$ sec and the damping ratio will be $\zeta = 1/\sqrt{2}$. Draw a root locus of the PD control system, and graphically obtain both the proportional gain *K* and the derivative gain k_D that meet the specifications.

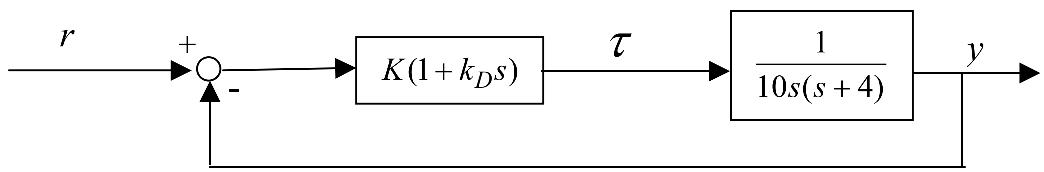

Figure 3 Proportional-plus-Derivative control of the ladder end point *y*

(2-c). Now consider disturbance F_d acting on the PD control system designed in part (2-b). See Figure 4. For this system obtain the steady-state error for a unit step disturbance F_d as well as the one for a unit step reference input *r*.

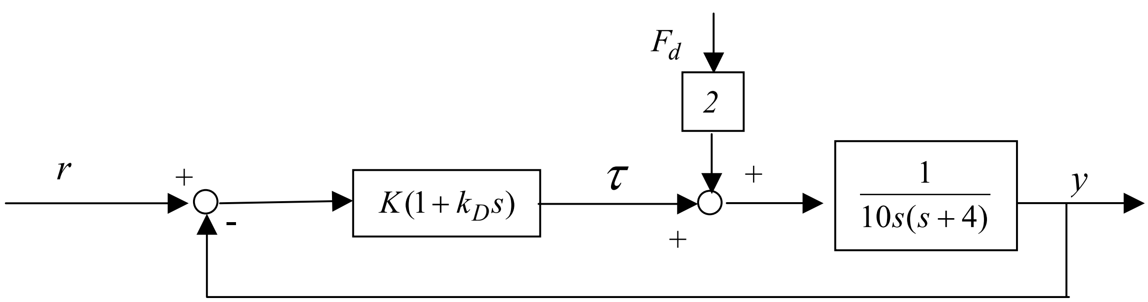

Figure 4 PD control system with disturbance F_d

(2-d) Design a cascade dynamic compensator that meets all of the following specifications:

- The steady-state error for a step disturbance is ***zero.***
- The steady-state error for a unit ramp disturbance is 6.25 %.
- The transient response that has been improved in part (2-b) will not significantly be changed.

Obtain the poles and zeros of the combined dynamic compensator including the above PD control and this new compensator. Draw the root locus of the total system.