Massachusetts Institute of Technology
Department of Mechanical Engineering
2.010 Modeling, Dynamics, and Control III

Quiz #3

May 9, 2002
3:00 pm – 4:30 pm

Close book. Two sheets of notes and a calculator are allowed.
Show how you arrived at your answer.
Do not detach Bode plots attached to your test booklet.

Problem 1 (70 points)

Consider a linear system whose open-loop frequency response is given by Figure 1, the Bode plots attached to your test booklet.

(1-a). What is the type of the system? What is the relative order, i.e. the number of poles minus the number of zeros? What are break frequencies? Explain how you found them.

(1-b). Find the gain margin and phase margin of the system. When the feedback loop is closed with a unity feedback gain, what is the undamped natural frequency of the closed-loop system?

(1-c). To improve stability and transient response, the following lead compensator has been proposed:

$$G_c(s) = \frac{s+1}{0.1s+1}$$

Sketch the gain and phase curves of this compensator on the blank log-log sheet attached to the test booklet. Note that the maximum phase angle is 55 degrees.

(1-d). On the Bode plots in Figure 1, sketch the new gain and phase curves of the open-loop control system compensated with the above lead compensator. Obtain the new gain margin and phase margin of the compensated system.

(1-e). Find the approximate percent overshoot and peak time of the compensated system.

(1-f). To improve steady-state error characteristics, a lag compensator with a zero frequency gain of *10 dB* has been added to the system. The break frequencies of the lag filter have been placed far from the phase margin frequency so that the system's transient response may not be altered. Using the attached Bode plots, graphically obtain the velocity constant when this lag compensator is used.

Problem 2 (30 points)

Consider a precision machine of mass $M=100$ *kg* suspended by a shock absorber of spring constant $K=10{,}000$ *N/m,* as shown in Figure 2-a. The shock absorber has some damping B, but the damping constant is unknown. Answer the following questions.

(2-a). When a step disturbance force F was applied to the mass, the percent overshoot of the mass was 20 %. When a sinusoidal disturbance force acts on the mass, the mass resonates at certain frequencies. Find the maximum magnitude of the resonant vibration and its frequency.

(2-b). To reduce vibration, a dynamic absorber consisting of mass m_a and spring constant k_a was added to the body of the precision machine, as shown in Figure 2-b. Using parameters M, B, K, m_a, and k_a, obtain the transfer function from disturbance force F to displacement y of the mass M, and find the finite disturbance frequency at which the magnitude of the displacement y becomes minimum. When $m_a=5$ *kg,* find the spring constant k_a such that the vibration magnitude may be minimum for a disturbance frequency of *10 rad/sec.*

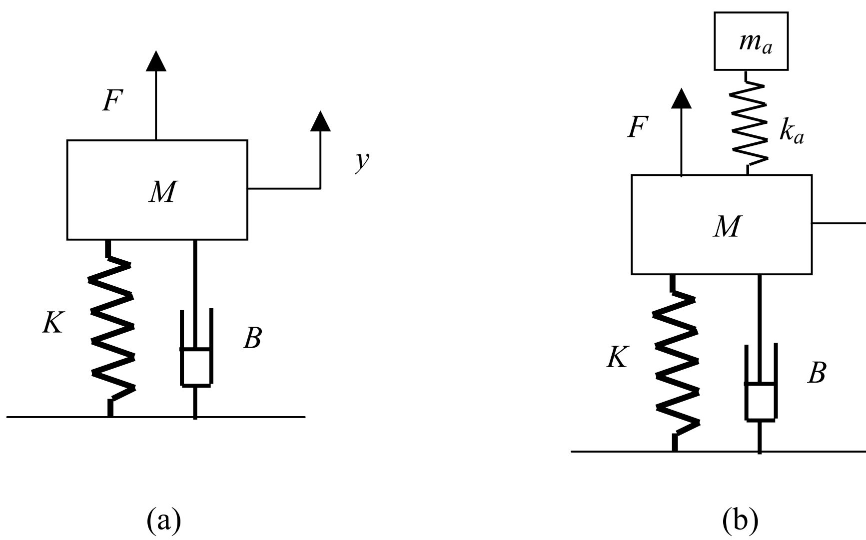

Figure 2