

Department of Mechanical Engineering

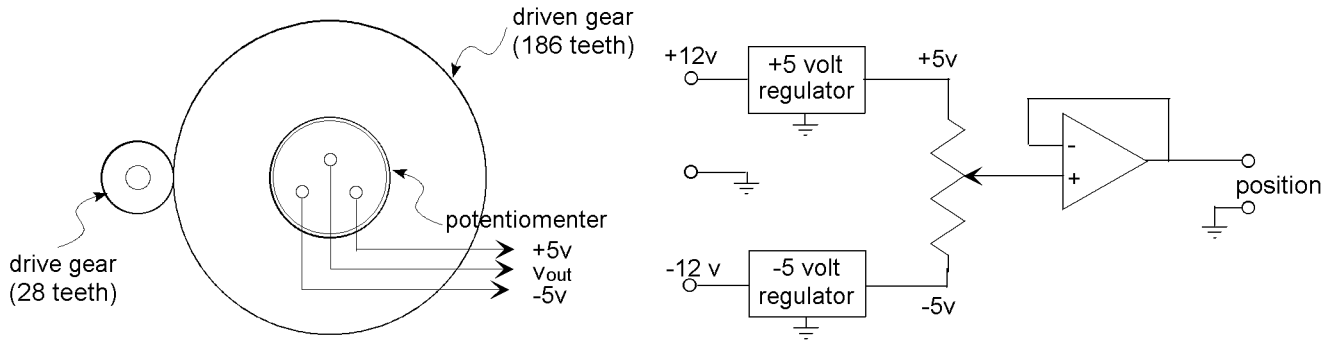
2.010 CONTROL SYSTEMS PRINCIPLES

Laboratory 4: Position Control of an Inertial Plant

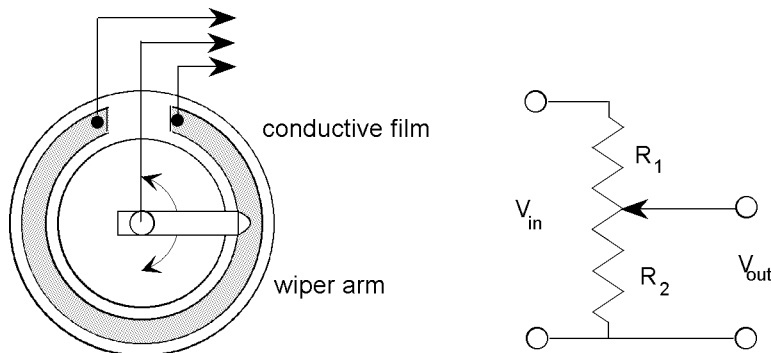
Laboratory Objectives: In Laboratory 3 you investigated proportional control of the angular velocity of a simple first-order plant: a rotary inertia with low damping. In this session we extend this work to the control of the *angular position* of the plant. In particular the goals are:

- (i) To investigate proportional position control of an inertial plant with negligible damping.
- (ii) To investigate the use of velocity feedback to add damping to the closed-loop system.
- (iii) To measure the steady-state disturbance rejection of the system.

The Position Sensor: Position control requires a *displacement sensor*. The laboratory motor/tach has an attached potentiometer to measure angular displacement, as illustrated below:



The potentiometer is driven through a step-down gear train. An internal electronics board contains a pair of 5 volt voltage regulators and a unity-gain buffer amplifier to prevent any attached circuitry from “loading” the potentiometer. The potentiometer consists of a uniformly distributed conductive film on a substrate with a wiper arm that moves across the surface.



The output voltage (if no current is drawn) is defined by the voltage divider

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

But $R_1 + R_2$ is constant, and R_2 is proportional to the shaft angle, therefore the output voltage is proportional to the shaft angle.

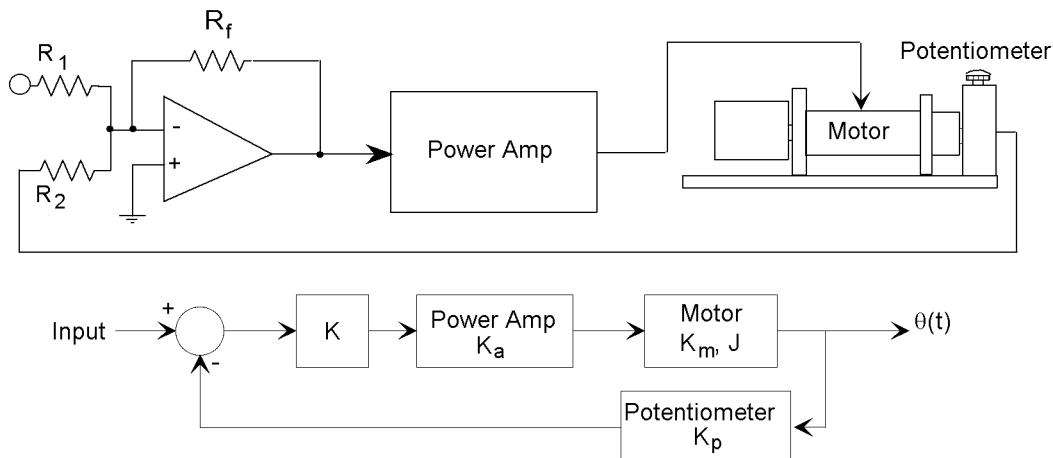
The potentiometer is of the *continuous rotation* type - that is it has no mechanical stops to limit the travel - with the result that the voltage is discontinuous as the wiper arm traverses the gap between the ends of the conducting film. As the shaft is rotated in the positive direction, the voltage increases to its maximum value (+5 volts), then for several degrees of rotation it is disconnected (there is no conducting film beneath it), and then suddenly it touches the negative end of the film and the voltage drops to -5 volts. The result is a sawtooth output voltage that repeats every rotation of the shaft. When we incorporate this sensor into a closed-loop system we have to be careful not to overstep the ends of the film.

Note: The potentiometer is engaged to the motor shaft by loosening the two knobs on the top of the plastic box and sliding the unit toward the shaft.

Step 1: Characterize the Position Sensor: The gain of the position sensor K_p (V/rad) should be determined by the following steps:

1. Connect the Wavetek DMM to the potentiometer's BNC connector and turn on the power to your breadboard. (The same power supply is used by the potentiometer circuit).
2. Rotate the shaft until the voltage reads zero, put a marker on the load at the top (or bottom) of the inertia.
3. By hand rotate the shaft in intervals of π radians (1/2 turn) and record the output voltage. Repeat this for the full range of the output voltage.
4. Use an Excel spreadsheet to plot the relationship between angle and voltage. K_p is the slope of the line.

Step 2: Construct a Proportional Position Controller:



Connect the op-amp controller that you built for Laboratory 3 (with a gain $K = 10$) as a position controller by using the potentiometer output instead of the tachometer, as shown above.

1. Measure the step-response by applying a small (≈ 0.25 V) square wave with a frequency of ≈ 0.2 Hz and monitoring the position on the oscilloscope.
2. Print a copy of the step response. Compare the form of the response with that predicted from the pre-lab exercise. Explain any differences. Is the envelope of the decay what you would from a linear system?
3. Your prelab exercise should have predicted a closed-loop undamped natural frequency

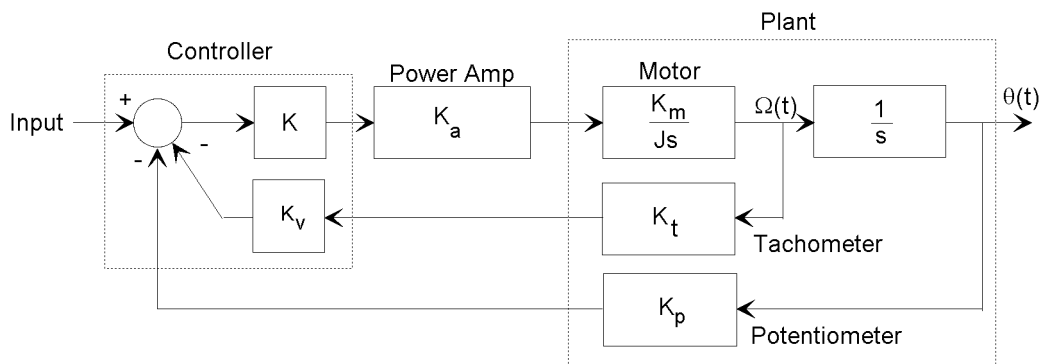
$$\omega_n = \sqrt{\frac{K K_a K_m K_p}{J}}$$

Use the values of the system parameters to compare the measured and theoretical undamped natural frequency of the closed-loop system.

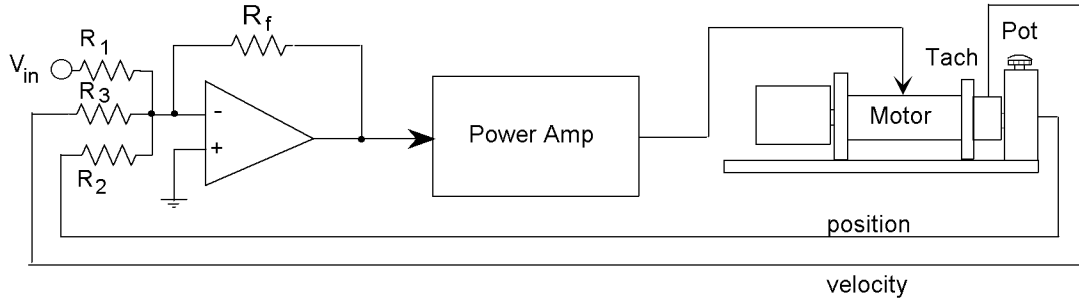
Repeat the procedure with amplifier gains of $K = 5$, and $K = 2$. Make a plot of the measured and predicted values of ω_n as a function of K .

Comment on the suitability of proportional control for this application.

Step 3. Proportional Control with Velocity Feedback: In the prelab exercise you were asked to derive the closed-loop transfer function for the system



We will investigate the dynamics of this modified form of the controller more thoroughly in Lab. 5. In this session we simply set up the controller and investigate its disturbance rejection. Modify your op-amp controller by adding an extra resistor, R_3 , to include the tachometer output in the sum, as shown below:

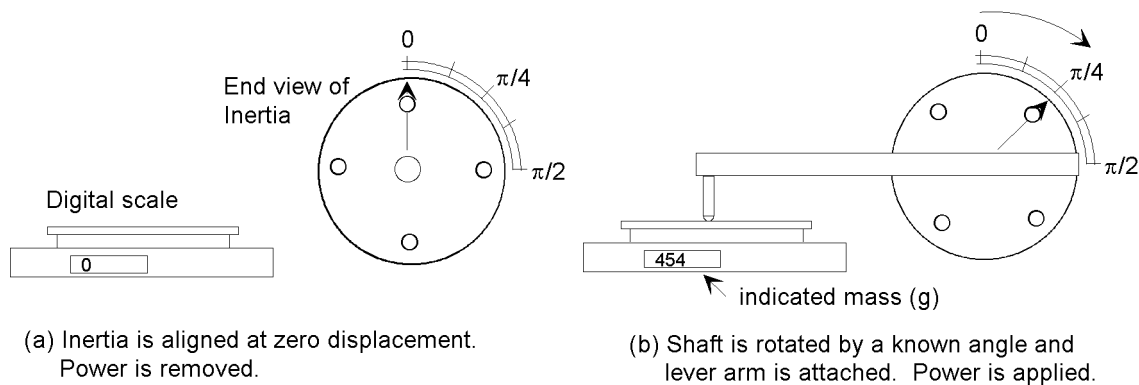


so that the system has both position and velocity feedback. Let $R_f = 100 \text{ k}\Omega$, $R_1 = R_2 = 10 \text{ k}\Omega$, and $R_3 = 22 \text{ k}\Omega$. Connect the tachometer to the new input and record the step response. Has it changed from that measured in Step 3? What has happened to the damping ratio ζ ?

Step 4. Measure the Steady-State Disturbance Rejection: With the velocity feedback connected, set the input voltage.

Use the lever-arm/digital scale to measure the restoring-torque/angular-displacement characteristic as follows:

1. Set the input voltage V_{in} to zero and allow the system to come to rest. Turn off the power amplifier at the circuit breaker.
2. Using the angular scale provided, rotate the shaft clockwise by a known angle θ_d (start with $\pi/6$ rad) and attach the lever arm assembly so that it rests on the digital scale. Check that the angle has not changed - we now have created a known angular displacement.
3. Zero the digital scale and turn on the power amplifier. Record the indicated “mass”, representing the restoring torque T_d .
4. Turn off the power amp.
5. Repeat these steps with the value of $R_f = 50 \text{ k}\Omega$, and $R_f = 22 \text{ k}\Omega$ so that you have measurements of the torque for controller gains of $K = 10, 5, 2.2$.



Repeat the whole set of measurements with the displacement set to $\theta_d = \pi/3$ and $\pi/2$ rad.

Use an Excel spreadsheet to make a plot of the relationship between torque T_d and displacement θ_d at each of the three gains, and from these curves compute the equivalent

motor shaft stiffness at each gain. Then make a plot of the variation of stiffness with the gain K .