

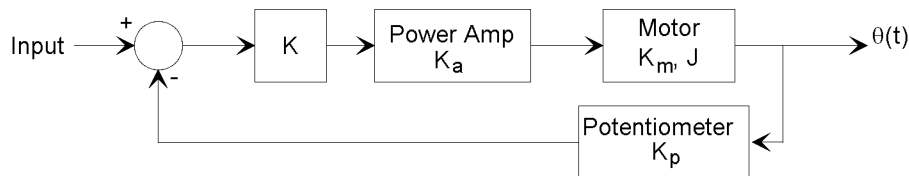
Department of Mechanical Engineering  
 2.010 CONTROL SYSTEMS PRINCIPLES  
Laboratory Session 4: Preparation Exercise

**Laboratory Session Objectives:**

- (i) To investigate proportional position control of an inertial plant with negligible damping.
- (ii) To investigate the use of velocity feedback to add damping to the closed-loop system.
- (iii) To measure the steady-state disturbance rejection of the system.

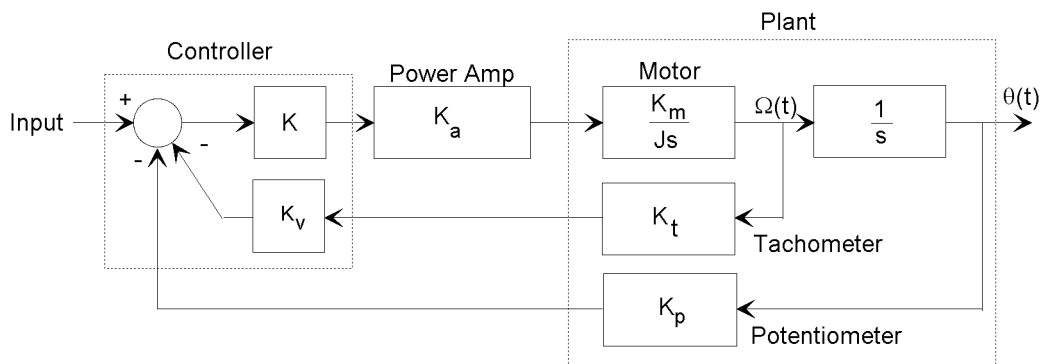
This exercise is intended to help you understand the measurements you will make. Bring you answers to the lab, and hand them in with your report.

**Exercise:** Consider the proportional *position* control system shown in the following figure:



- (a) Derive the transfer function relating the angular position  $\theta$  of the motor shaft to the voltage input command in terms of the controller gain  $K$ , servo amp gain  $K_a$ , motor torque constant  $K_m$  and inertia  $J$ , and position sensor gain  $K_p$ . Ignore nonlinearities such as Coulomb friction.
- (b) Derive the closed-loop undamped natural frequency  $\omega_n$  and damping ratio  $\zeta$ . Make a sketch of the expected form of the system's step response.

Now consider the addition of an inner velocity loop using the tachometer:



- (c) As above, derive the transfer function relating the angular position  $\theta$  of the motor shaft to the voltage input command. (Hint: Derive a transfer function for the inner loop first.)
- (d) Derive the closed-loop undamped natural frequency  $\omega_n$  and damping ratio  $\zeta$ .