Department of Mechanical Engineering

2.010 CONTROL SYSTEMS PRINCIPLES

Laboratory 9: Introduction to System Identification

Introduction:

In many practical situations you will not be able model a plant from first principles – the process is simply too complex, or the internal structure is hidden from you. In such cases you must make measurements on the system and infer a model (in the form of a transfer function) from your measurements. This process is known as *system identification*. The process usually involves two steps:

1. Determining a suitable structure for the model; for example deciding the system order and determining the order of the numerator polynomial.

2. Determining numerical values for the coefficients of the numerator and denominator coefficients from experimental measurements made on the plant.

In this laboratory session you will investigate four common methods of identifying a low-order plant using (i) the step response, (ii) the impulse response, (iii) the frequency response, and (iv) stochastic (or random) inputs.

Setting Up the Model:

Set up for the session as follows:

1. Construct a unity-gain proportional position controller using a single op-amp as shown below:
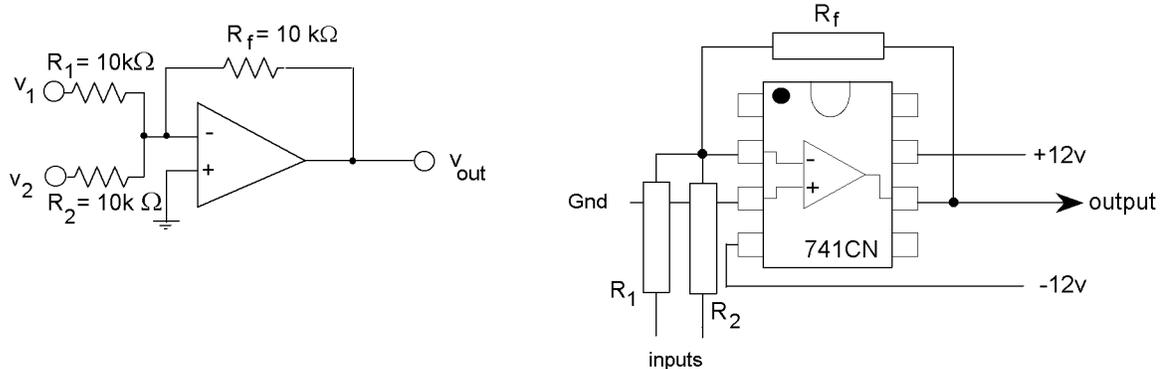
   You will use the closed-loop system as the plant to identify.

2. Assume that the closed-loop system is a classic second-order system with no zeros, that is it has a transfer function of the form:

$$G_{cl}(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The task is to identify the three parameters, the steady-state gain K, the damping ratio ζ, and the natural frequency ω_n. The first question you should ask is – is this a valid model? You should answer the question by deriving the closed-loop transfer function (if you haven't already).

3. You will use three methods of system identification; (i) the impulse response, (ii) the step response, and (iii) using random waveforms. In each case you will excite the system with the waveform, monitor the response and compare the response to a second order system.

Parameter Estimation:
In the `Lab9` folder in the `2.010` folder on the computer desktop you will find a program `SecOrdSys`. This program is a convenient means for displaying second-order responses; it allows you to adjust the parameters K, ζ, and ω_n and display the system responses. Start the program and experiment with it. In the following measurements you will use the program to match your measured responses with the theoretical responses.

System Identification by Impulse Response:
The Dirac impulse function $\delta(t)$ is defined to be a pulse of infinitesimal duration and infinite amplitude:

$$\begin{aligned} \delta(t) &= 0 \qquad t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)dt &= 1 \end{aligned}$$

where $\delta(t)$ is strictly undefined at $t = 0$. The impulse is a mathematical construct – it cannot be generated in practice, but it can be approximated by a short duration pulse with unit area. You can use the function generator to generate a short pulse waveform of amplitude A and duration T and assume it to be an impulse of strength AT.

Excite the system with an impulse-like pulse and monitor the response with the oscilloscope. Use `SecOrdSys` to find a set of system parameters that match your impulse response and record them.

System Identification by Step Response:
Use the function generator to generate a unipolar step function. Monitor the response and use `SecOrdSys` to match the parameters.

Frequency Response Measurement Using Random Noise
In this section you will measure the system frequency response by another method. The idea is to excite the system with a *random* input—i.e., white noise—and to measure the spectral relationships of the input and output. In 2.003 we examined briefly the Fourier transform of non-repetitive wave forms. The Fourier methods decompose a waveform $f(t)$ into spectrally weighted components described by the Fourier spectrum $F(j\omega)$. When a waveform is passed through a linear system, the spectrum of the output $Y(j\omega)$ is

$$Y(j\omega) = H(j\omega)U(j\omega)$$

where $H(j\omega)$ is the frequency response function and $U(j\omega)$ is the spectrum of the output. Thus, in principle $H(j\omega)$ can be determined by exciting a system with an input waveform

that is "rich" in frequencies, and taking the ratio of the spectra of the output and input. In practice the analysis used is more complex to make the measurement more robust (it involves taking the ratio of the input-output *cross-power spectrum* to the input *auto-power spectrum*), but the principle is the same.

The input waveform should be as "white" as practicable, that is it should contain a uniform distribution of power across its spectrum. (The term "white" is by analogy to white light – which contains an equal distribution of all spectral colors). A random waveform (made up of a set of random numbers) is a "white" source. In this experiment we have used the `RAND()` function in Microsoft's Excel to generate a sequence of uniformly distributed 2048 random numbers. We recorded them into a file 'random.txt', as a *user-defined* waveform for the function generator. We will use the Virtual Bench *Digital Signal Analyzer* (DSA) to record both the input waveform and the system response to compute the frequency response function in real-time.

The DSA is an instrument that performs sophisticated analyses on experimental time-series data records. It has a direct built-in capability to determine the frequency response function using random excitation. The instrument is somewhat difficult to set up; if you have problems your lab instructor will help you.

(1) Start the function generator and load the random waveform as follows. On the menu File/Load Waveform... you will find a file `RANDOM.TXT`, click on it, and then click on the OK button in the preview. Now you can invoke the file by selecting `USER` on the FG front panel. We will run the waveform at a low frequency (≈ 0.1 Hz). Test your system with the waveform.

(2) Start the Virtual Bench *Digital Signal Analyzer* (DSA). We have prepared a configuration file that will set up the analyzer for today's experiment. On the Configuration/Load Settings.. menu select the file `LAB9.SET`. You should be ready to run.

(3) Turn on the DSA. Make sure that the output of the function generator is connected to Channel 1 on the input/output box, and the output of the potentiometer is connected to Channel 2. When you are sure that the DSA is displaying the magnitude and phase of the frequency response, print it out. Use `SecOrdSys` to estimate the system parameters. **Include plots of the frequency, step and impulse responses in your report.**