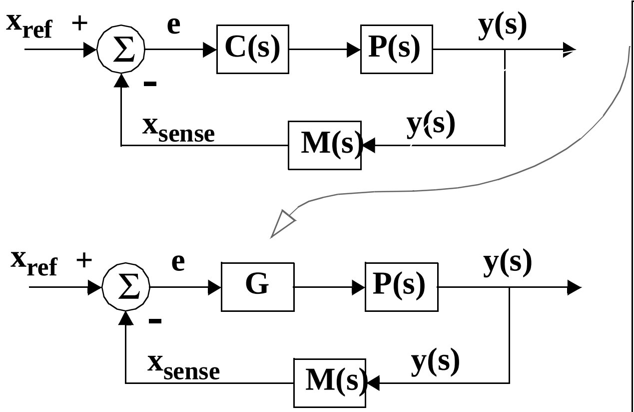
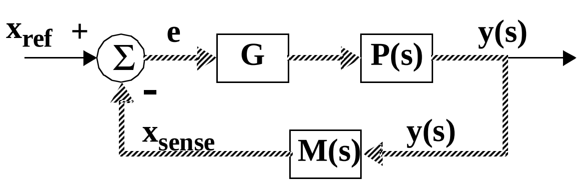
2.010: Comments on Proportional (P) Controller Analysis (HW #3) [Oct-5-2000]

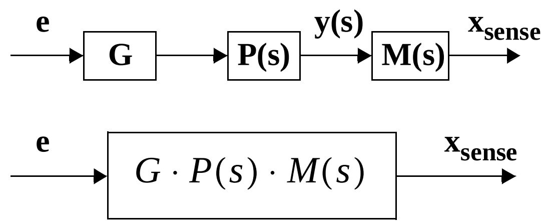


For a "proportional" feedback controller, **"C(s)" is just "G" (a constant)**. The resulting control system simply multiplies the "error" [$e = x_{ref} - x_{sense}$] by this constant value, "G", to command the Plant.

- •C(s) : The Controller
- •P(s) : The Plant (system being controlled)
- •M(s) : Your Measurement (often, M(s)=1)
- •e : The error
- •xref : a reference input
- •y(s) : The system output

- Now, let's look at the "open-loop" transfer function (at left, below):





- The open-loop TF (or "loop transmission") is the transfer function you get by making one, complete circuit of the feedback loop, as shown above at right.

- Note it does not matter where you break the loop, as long as you make one, complete circuit, since: $G \cdot P(s) \cdot M(s) = P(s) \cdot M(s) \cdot G = M(s) \cdot G \cdot P(s)$

Stability vs. Performance...

- Often, there is a conflict between **accuracy** (greater accuracy/performance/speed by making G larger) and **stability** (greater stability by making G smaller).

- Some open-loops systems are inherently stable, since the phase of the open-loop transfer function nevers goes below $-180°$.

Criteria for stability

- For a system to be stable, we mean that when we give a "bounded-input" (finite input), we will get back a "bounded-output" (the system will not "blow up" exponentially!). [BIBO = "bounded-input, bounded-output"].

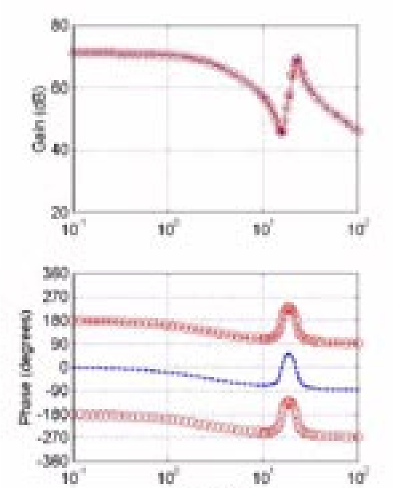
- Look at the solutions to the denominator of the transfer function. These eigenvalues are the system "poles". (Hitting the frequency of a "pole" tends to send the resulting system response "up", like a pole sticking out of the ground. Hitting the frequency of a "zero" tends to have the reverse effect, hence the names. To remember this, recall that setting the denominator of a fraction sends it to infinity, while setting the numerator to zero sends it to zero.)

- The resulting mode shapes are all exponentials:

$$A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + \ldots + A_n e^{\lambda_n t} = A_1 e^{p_1 t} + A_2 e^{p_2 t} + \ldots + A_n e^{p_n t}$$

When $Re(p_n) > 0$, we have exponential growth, and the system "blows up" over time. We say that the system is UNSTABLE. When $p_n = 0$, we have a term which neither grows now decays over time, and the result is *marginally stable.*

- A system is stable if and only if <u>all</u> system poles lie strictly in the left half plane [including NOT on the Imaginary axis itself) of the Re-Im axes.



Here is an example of an open-loop transfer function:

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Transfer function:
20000 s^2 + 4.8e004 s + 4.96e006
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 s^3 + 6.4 s^2 + 505.4 s + 1389
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It is plotted with the solid dots (and line) at left. If we call this system 'T(s)=G*P(s)*M(s)', then what can we say about the Bode plot transfer function for '-T(s)' (negative T(s))???

1. The magnitude is IDENTICAL. (Overlaps on upper plot)
2. The PHASE is off by exactly $180°$. (This identically either +180 or -180 from the original, as shown w/ plotted circles.)

Ways to test/look for system stability:

- Looking at the open-loop system Bode plot, **does the PHASE go below -180 degrees?** If so, make sure the GAIN is LESS THAN 0dB where the system goes to -180 degrees. (In other words, make sure you have a <u>positive gain margin</u>.) If not, then you have no worries about a 'negative gain margin' (since the system just nevers drops below -180 degrees in phase).

- Looking at the closed-loop system transfer function, **are the Real parts of ALL the system poles strictly negative?**

For a transfer function with higher order terms (in 's') in the denominator, it can be pretty tough to figure out if all the roots have negative real parts! ...That said, here are a couple of quick tricks:

1. Let us represent the FORWARD transfer function (G*P(s)) as "N(s)/D(s)". That is, some polynomial in 's' in the numerator, divided by some polynomial in 's' in the denominator.
2. For a measurement of '1', the closed-loop transfer function is:

$$\frac{\frac{N(s)}{D(s)}}{1+\frac{N(s)}{D(s)}} = \frac{N(s)}{D(s)+N(s)} \qquad \text{For example, if: } \frac{N(s)}{D(s)} = \frac{G\cdot(as^2+bs+c)}{(s^3+xs^2+ys+z)};$$

$$\text{Then: } \frac{N(s)}{D(s)+N(s)} = \frac{G\cdot(as^2+bs+c)}{(s^3+(Ga+x)s^2+(Gb+y)s+(Gc+z))}$$

***NOTE:** In order for ALL the roots of the denominator to be NEGATIVE (stable), **all powers of 's' must have coefficients of the SAME SIGN** (and none must be missing, i.e.no coef is 'zero').
<u>!!!-->Unfortunately, some polynomials satisfy this requirement but STILL have positive root(s)</u>, but if "Gc+z" is less than zero, you know immediately this system can NOT be stable.
(Since s^3 is (+1)*s^3, and +1>0, then: **Ga+x>0, Gb+y>0, Gc+z>0, all <u>required</u> here**.)