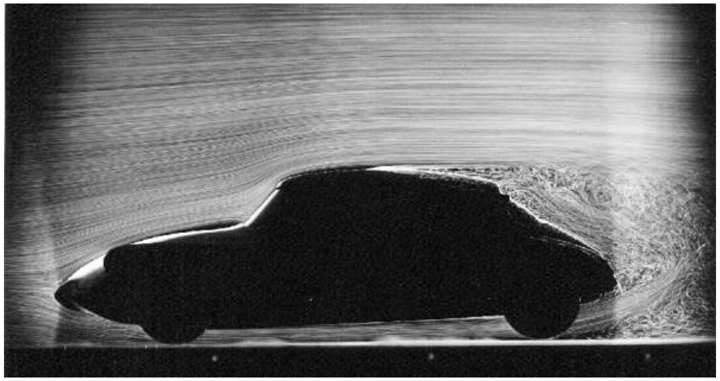
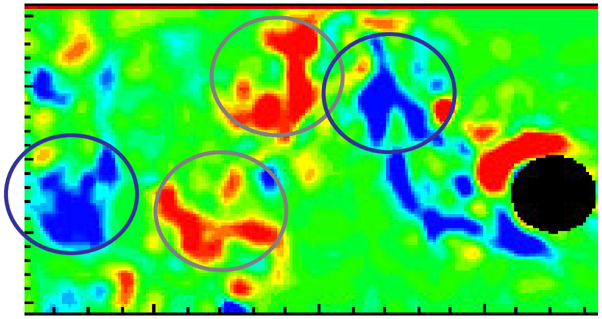
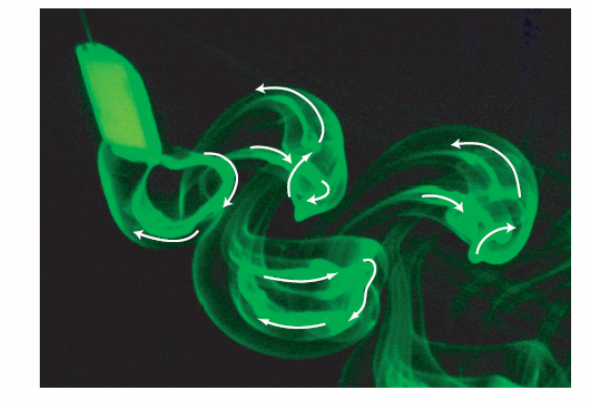




Separated Viscous Flows & Vortex Induced Vibrations

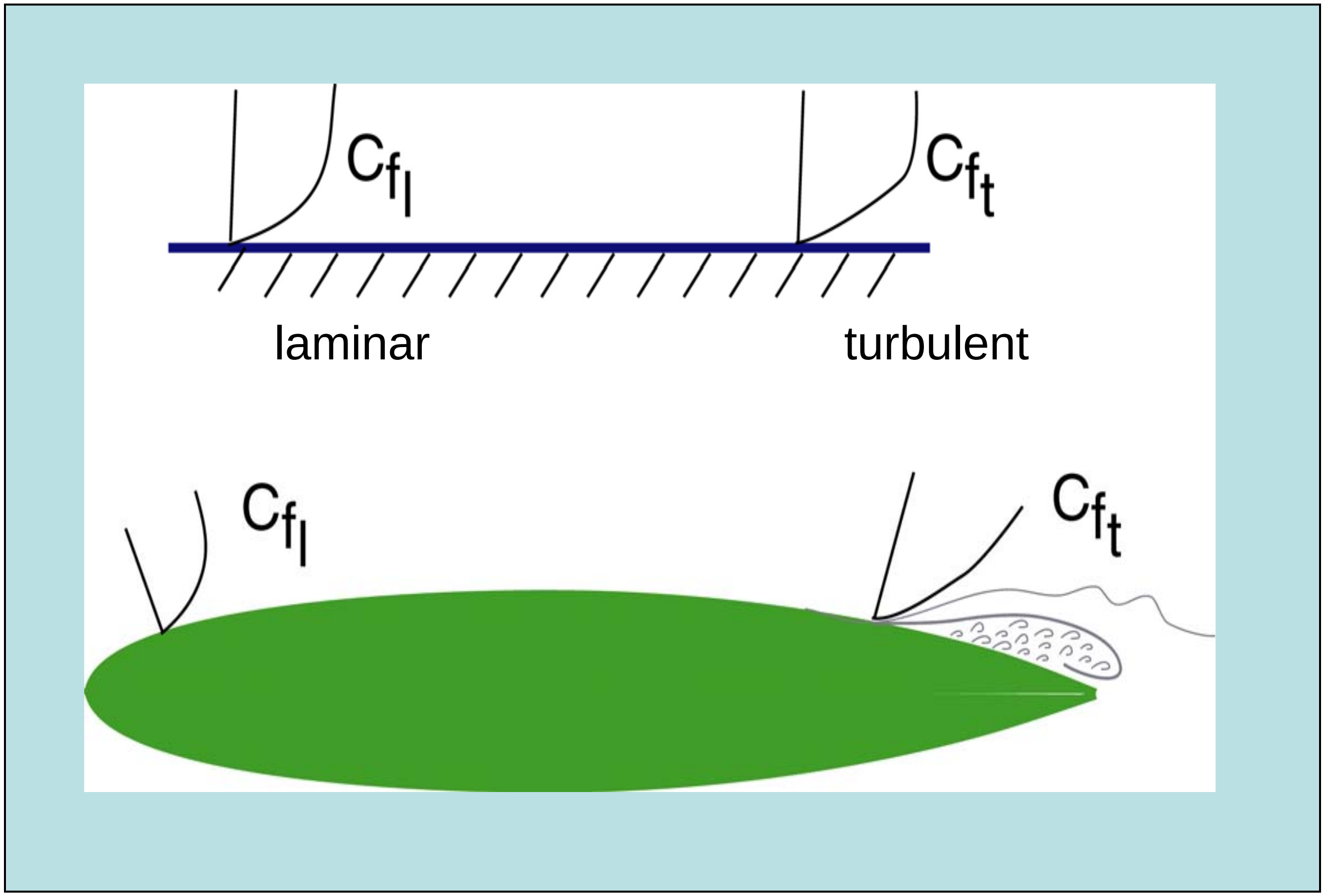


2.016

Fall 2005

Prof. A.H. Techet

Viscous Drag



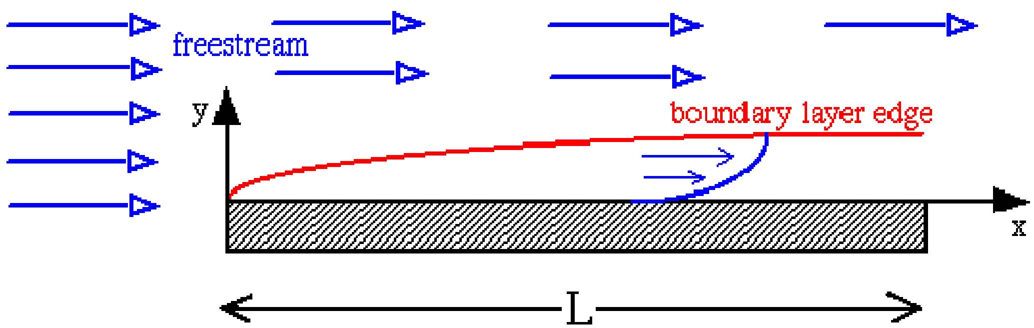
Skin Friction Drag: C_f

Form Drag: C_D due to pressure (turbulence, separation)

Streamlined bodies reduce separation, thus reduce form drag.
Bluff bodies have strong separation thus high form drag.

Boundary layer development

Flow over a flat surface causes a velocity gradient near the boundary due to the NO-SLIP condition.



NO SLIP CONDITION: requires that the velocity of the fluid at the wall matches the velocity of the wall, such that it does not “slip” along the boundary.

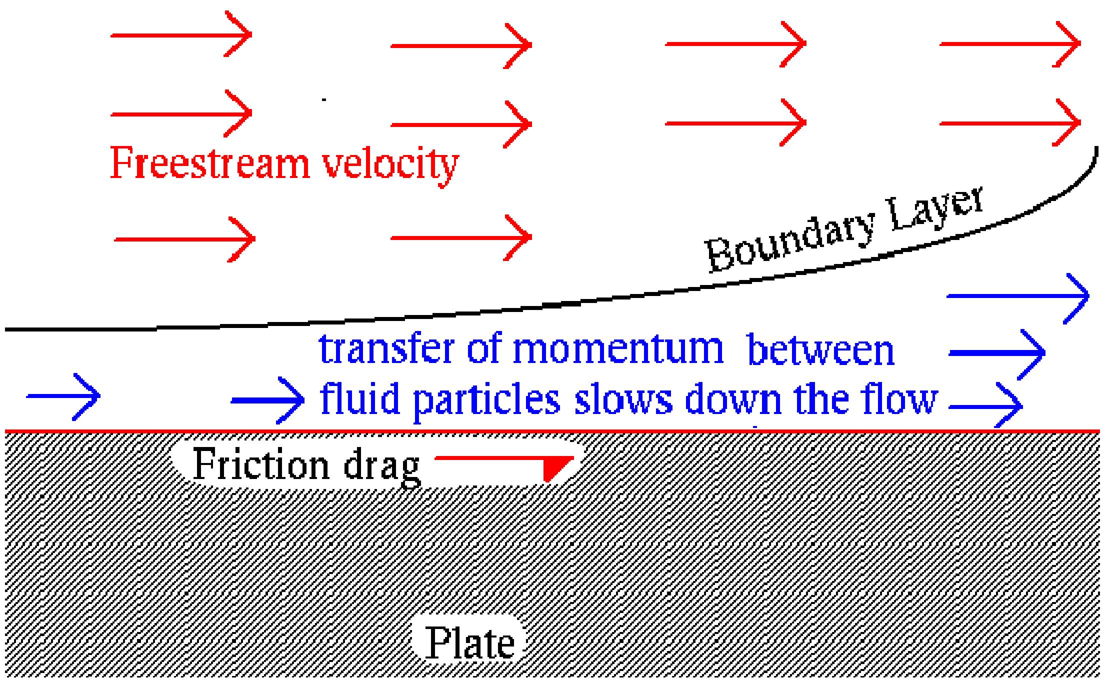
Since the velocity away from the wall is much faster a velocity gradient results in the boundary layer region since fluid is a continuous medium. In this region viscosity plays a strong role.

Wall shear stress:

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

Friction Drag

The transfer of momentum between the fluid particles slows the flow down causing drag on the plate. This drag is referred to as friction drag.

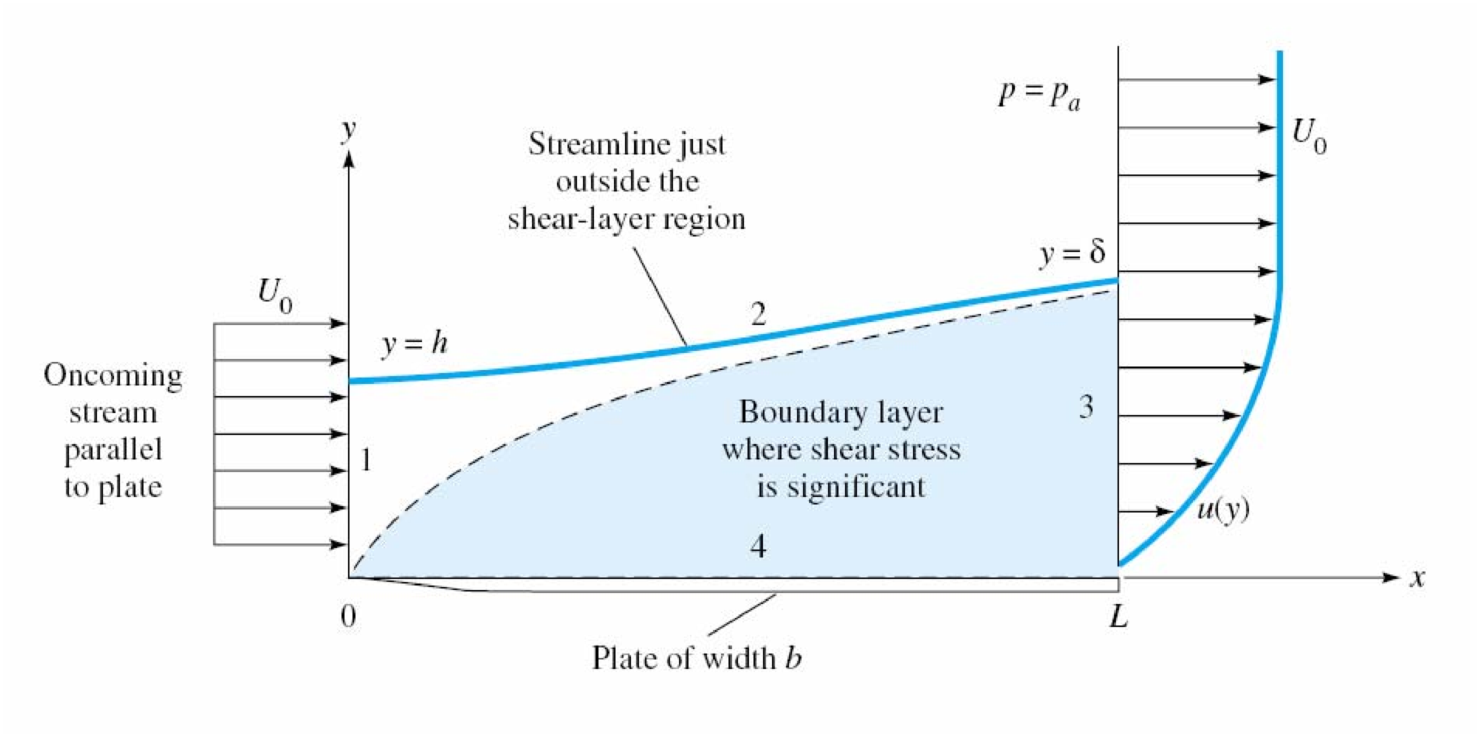


units

Friction Drag Coefficient:

$$C_f = \frac{F}{\frac{1}{2}\rho U^2 A_w} \qquad \frac{[MLT^{-2}]}{[MLT^{-2}]}$$

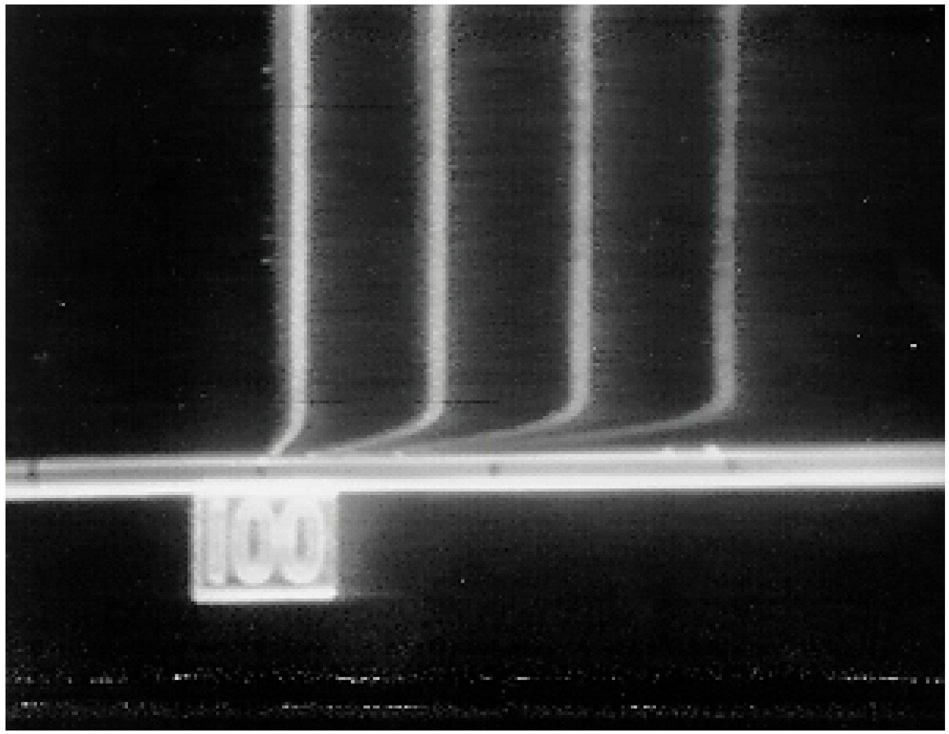
Boundary Layer



δ = boundary layer thickness

δ_{99} = BL Thickness where $u(y) = 0.99U_o$

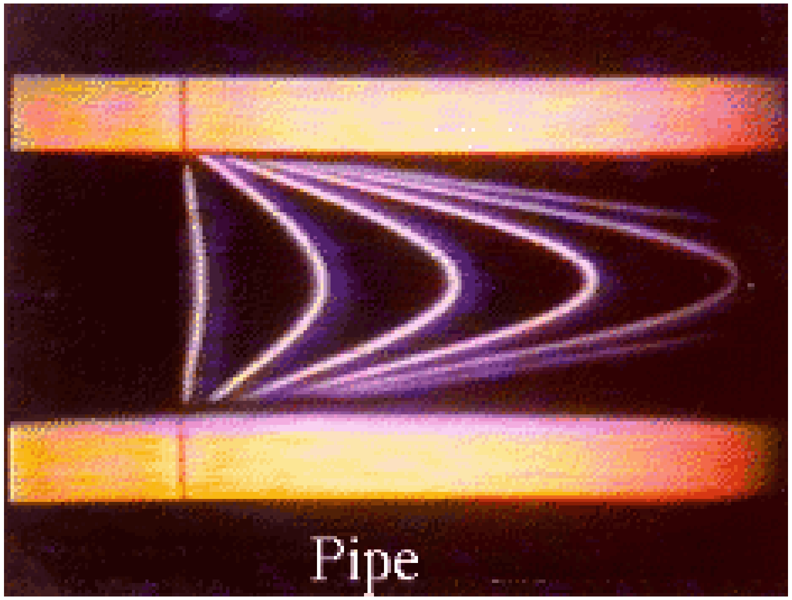
Boundary Layer Growth

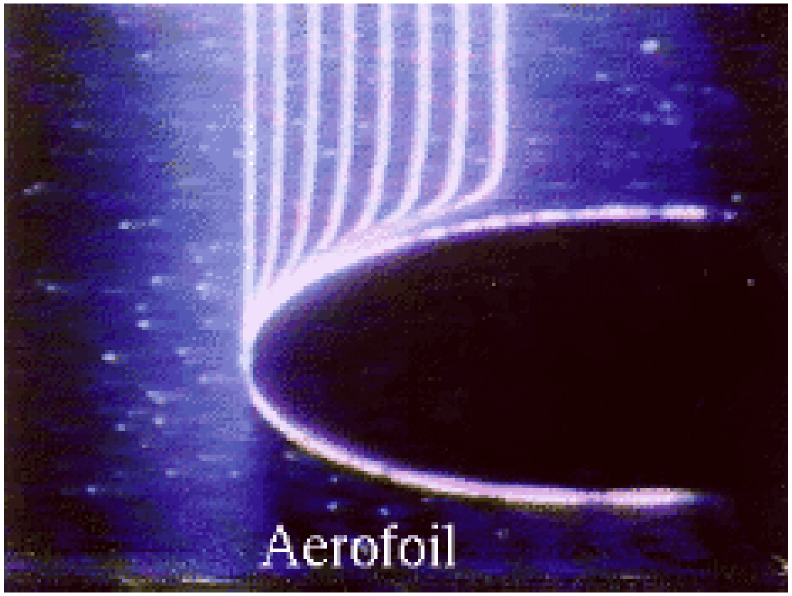


As a boundary layer on a plate grows its thickness increases with distance, *x.* The Reynolds number of a boundary layer is defined as

$$Re_x = Ux/\nu$$

Boundary layer growth



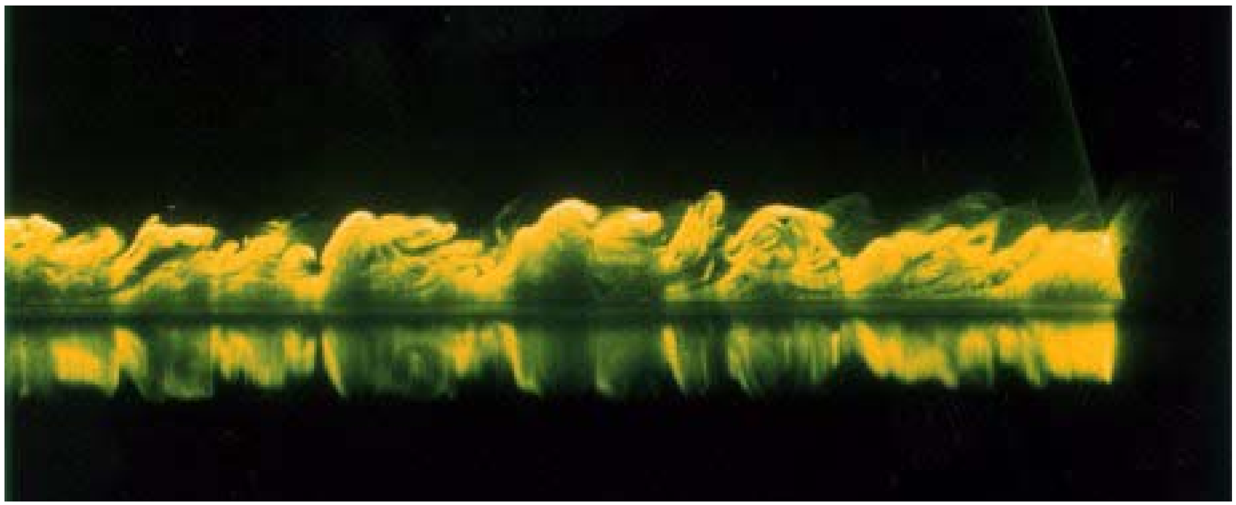


Boundary layers develop along the walls in pipe flow. A cross sectional view shows the layer at the top and the bottom creating a symmetrical profile. The flow in the middle is fastest since it has not been slowed by the momentum transfer in the boundary layers.

Boundary layers develop along the aerofoil shown above. The velocity profile changes shape over the curved leading edge. Towards the trailing edge the flow tends to separate as a result of an adverse pressure gradient.

(pictures courtesy of Iowa Institute of Hydraulic Research, University of Iowa)

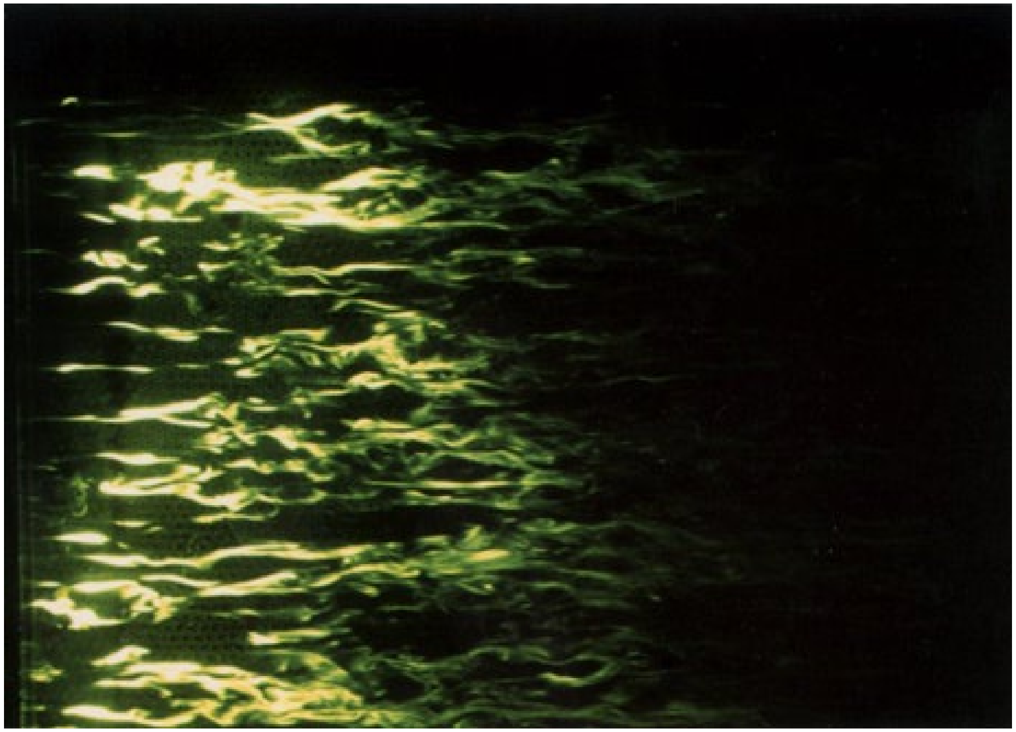
Turbulent Boundary Layers



This picture is a side view of the large eddies in a turbulent boundary layer. Laser-induced fluorescence is again used to capture the quasi-periodic coherent structures. Flow is from left to right.

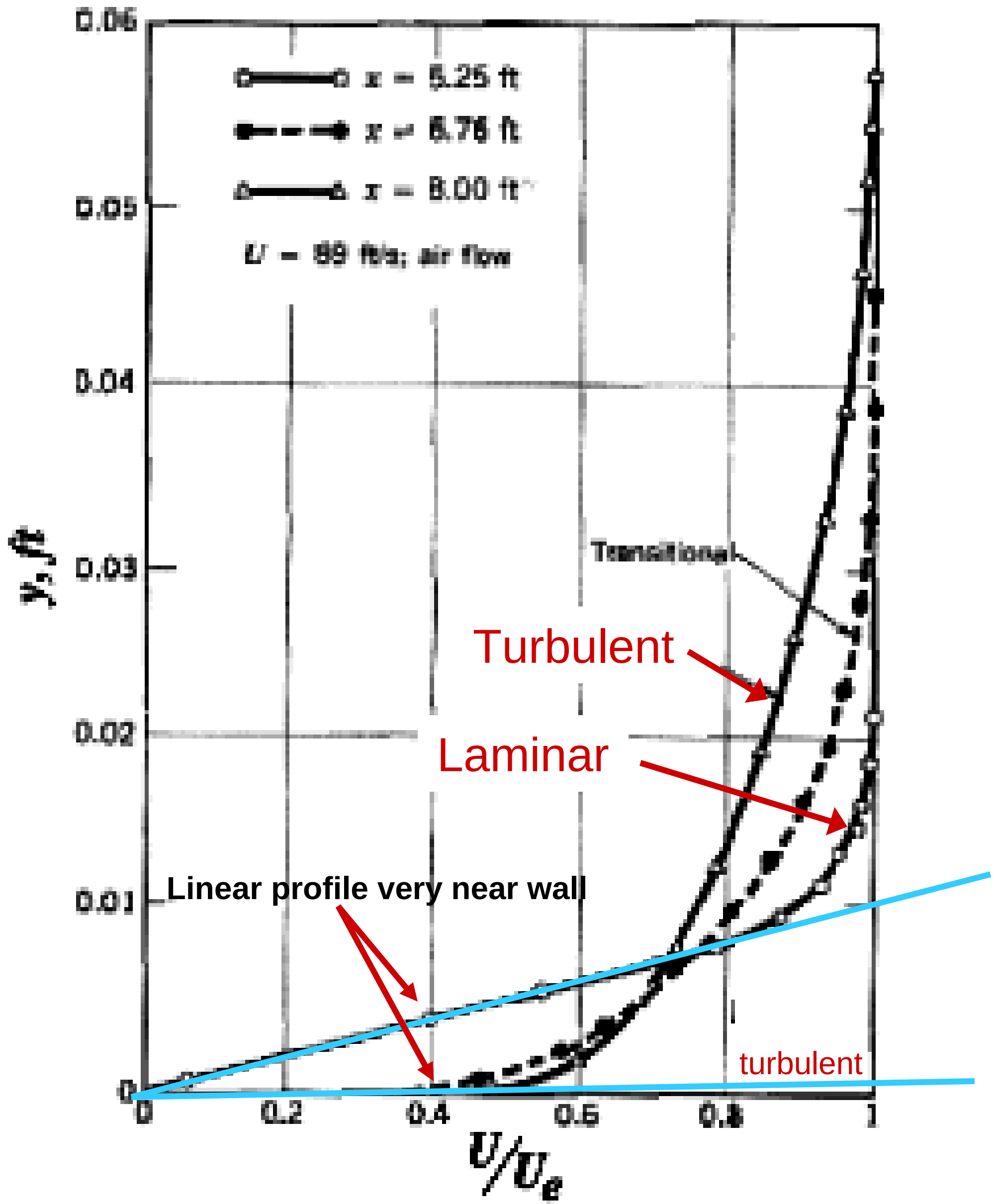
Contributor: Prof. M. Gad-el-Hak, University of Notre Dame

Turbulent Wall Boundary Layer (Top View)



This picture is a top view of the near-wall region of a turbulent boundary layer showing the ubiquitous low-speed streaks. Flow is from left to right and laser-induced fluorescence is used to visualize the streaks.

Contributor: Prof. M. Gad-el-Hak, University of Notre Dame



Turbulent vs. Laminar Boundary Layers:

It can be seen from these plots that the two boundary layers have quite different shapes.

Wall shear stress:

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

The <u>slope of the velocity</u> profile near the wall is less for a <u>laminar</u> boundary layer implying a decreased wall shear stress.

Flat Plate Friction Coefficient

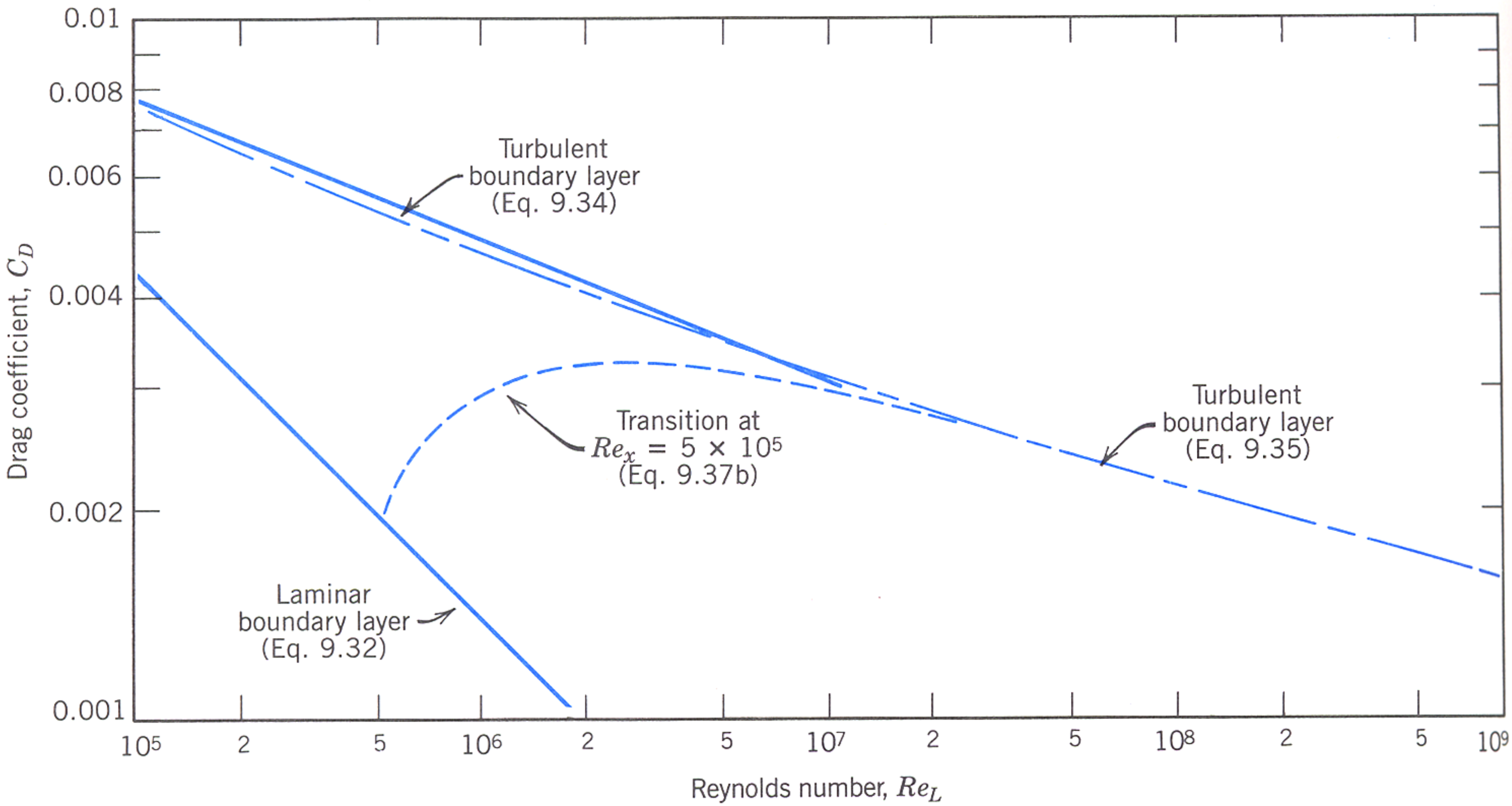
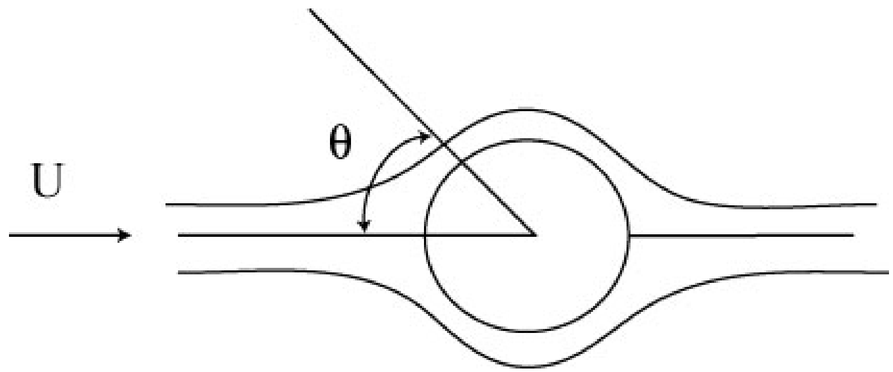


Fig. 9.8 Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.

Viscous Flow around Bluff Bodies (like cylinders) tends to separate and form drag dominates over friction drag.

Potential Flow → No Drag

Streamlines around the body are symmetric fore/aft (and top/bottom) thus there is no pressure differential that could result in a force on the body.



$U(\theta) = 2U_\infty \sin\theta$

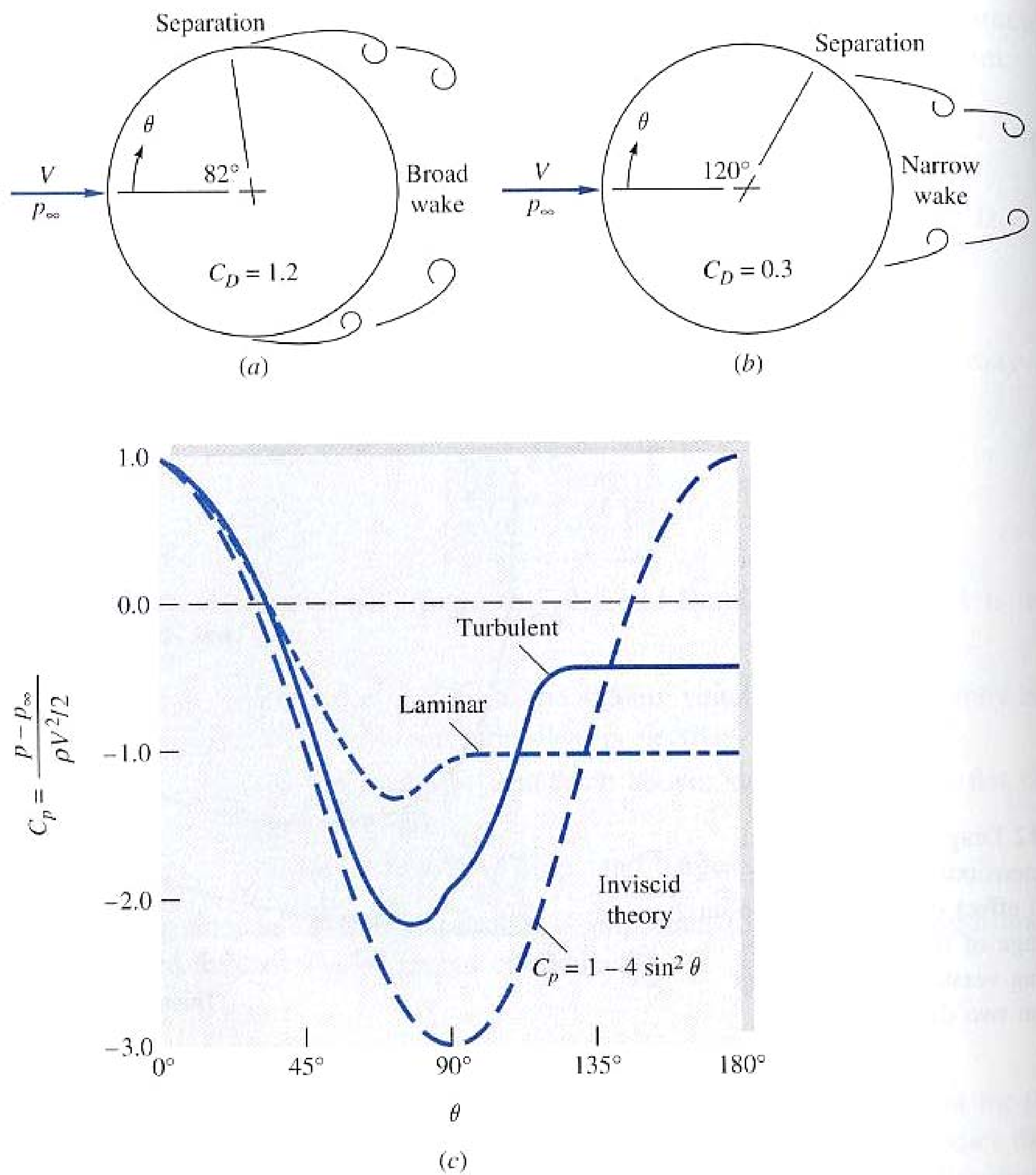
$P(\theta) = 1/2\ \rho\ U(\theta)2 = P_\infty + 1/2\ \rho\ U_\infty^2$

$Cp = \{P(\theta) - P_\infty\}/\{1/2\ \rho\ U_\infty^2\} = 1 - 4\sin^2\theta$

Pressure around a cylinder in viscous flow:

(a)Laminar

(b)Turbulent



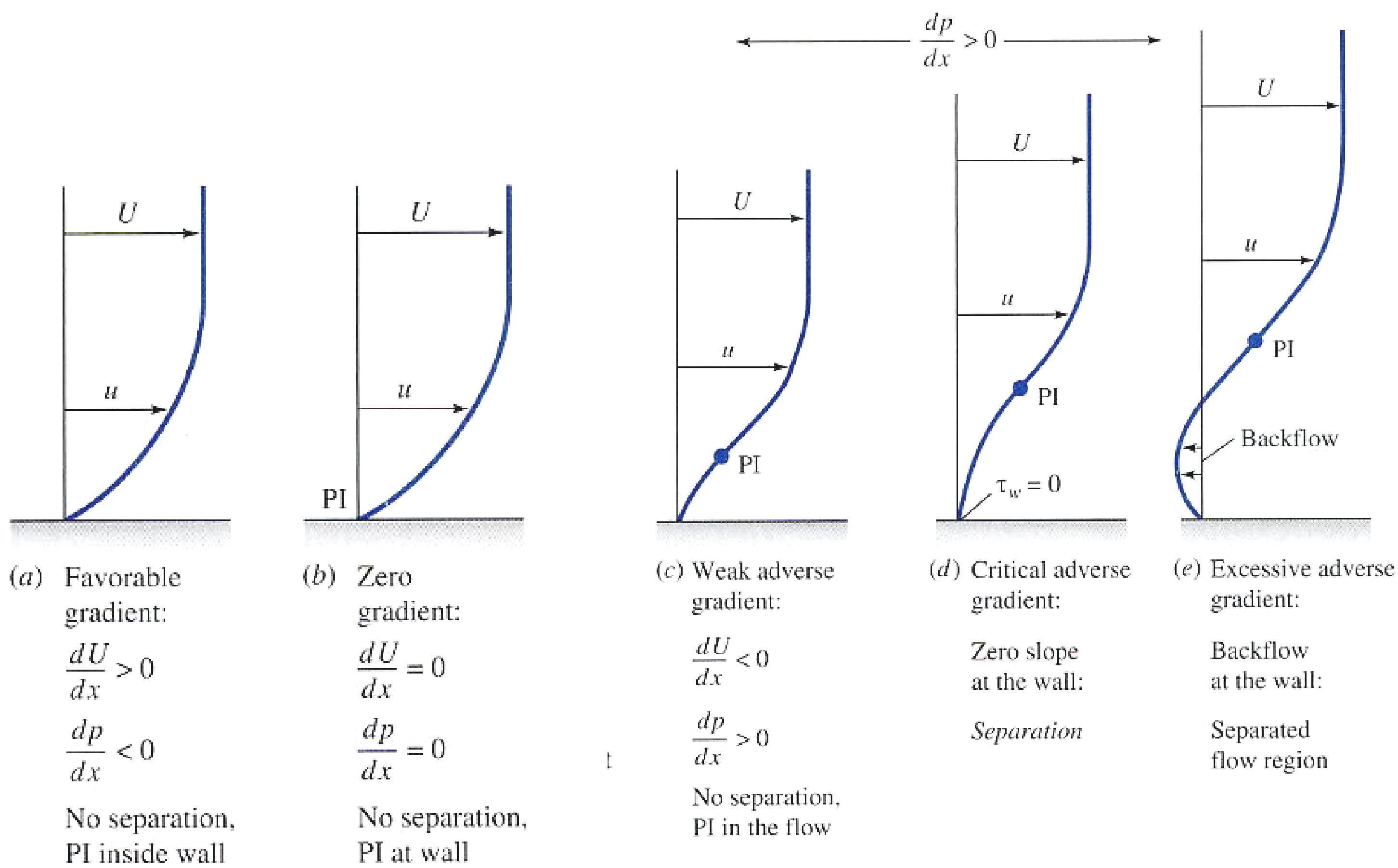
From F.M. White, Fluid Mechanics, 5th Ed. McGraw Hill 2005. p. 480

Viscous Flow → Form Drag

- Similar to the potential flow about a cylinder, in viscous flow there is a stagnation point at the leading edge where the pressure is high.
- From there the flow travels along the boundary and a boundary layer forms on leading edge of bluff body.
- As the flow moves around the body it is accelerated along the body and a FAVORABLE PRESSURE GRADIENT develops.
- Towards the separation point an ADVERSE PRESSURE GRADIENT develops.
- At the point of separation the wall shear stress is zero.
- Past the separation point the boundary layer profile shows reverse flow near the body indicating separation.
- The pressure in the near wake (or separation region) is lower than the stagnation pressure at the leading edge resulting in drag on the body.
- This drag is sometimes called

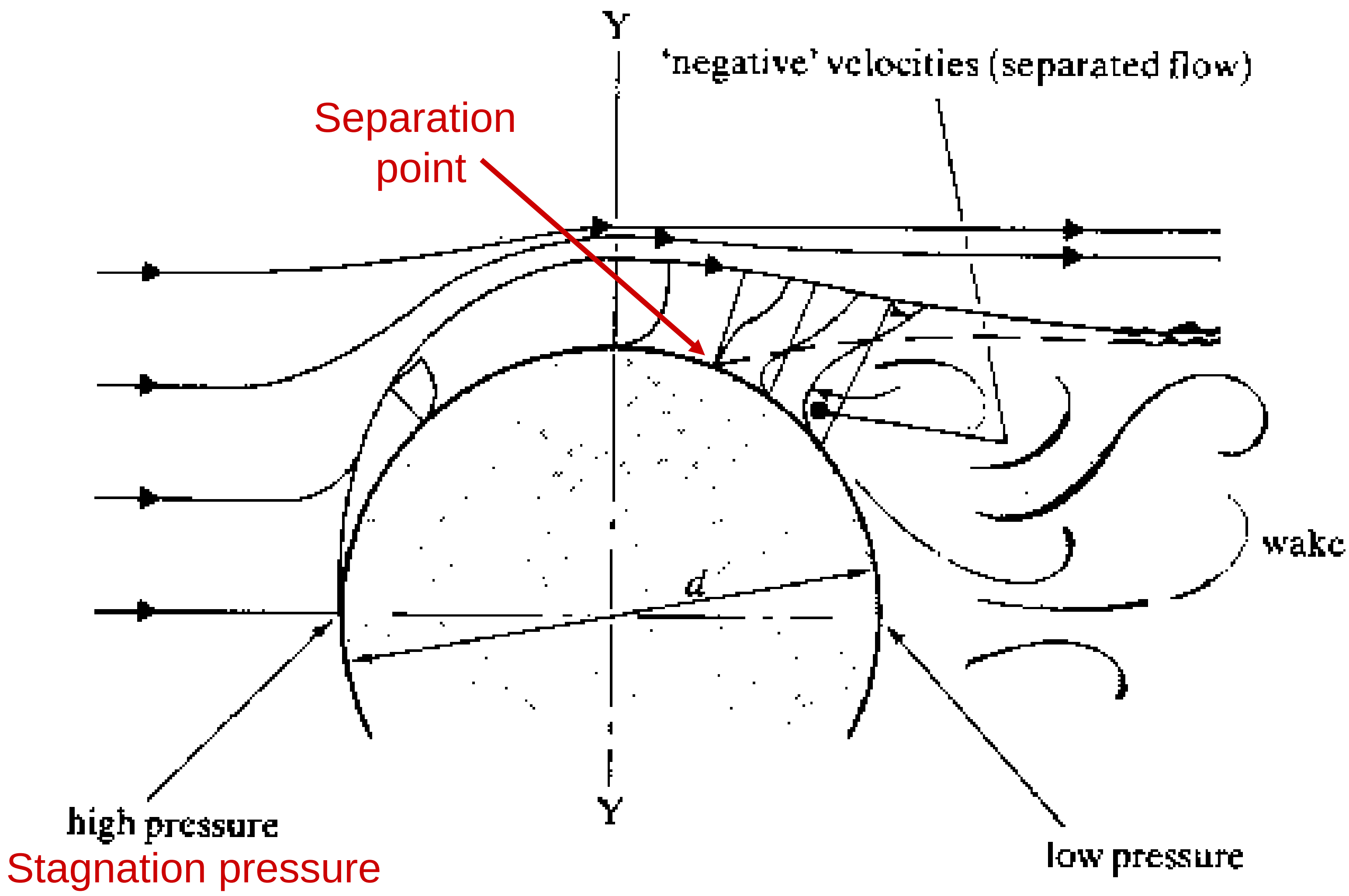
Form Drag or Separation Drag or Pressure Drag

Pressure Gradient Effects on Separation



From F.M. White, Fluid Mechanics, 5th Ed. McGraw Hill 2005.

Flow Separating from a Cylinder



Average Viscous Forces

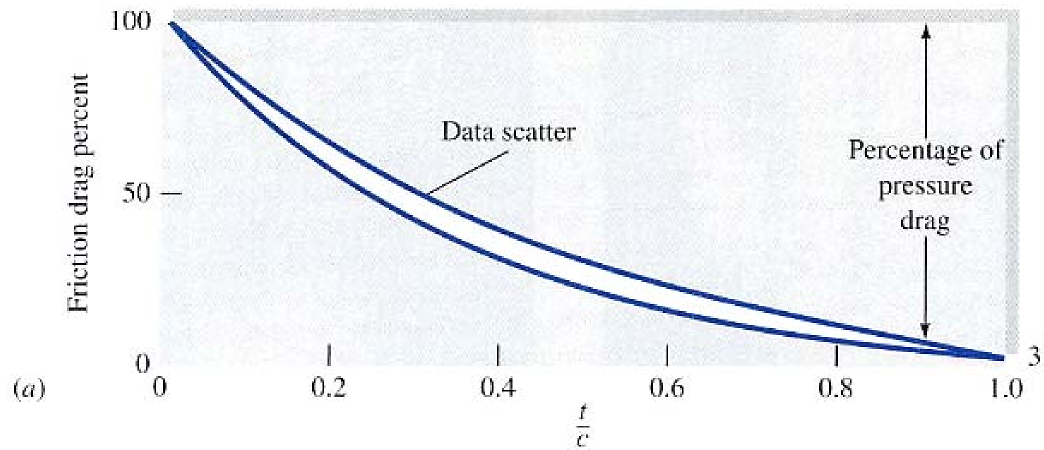
- Drag Force on the body due to viscous effects:

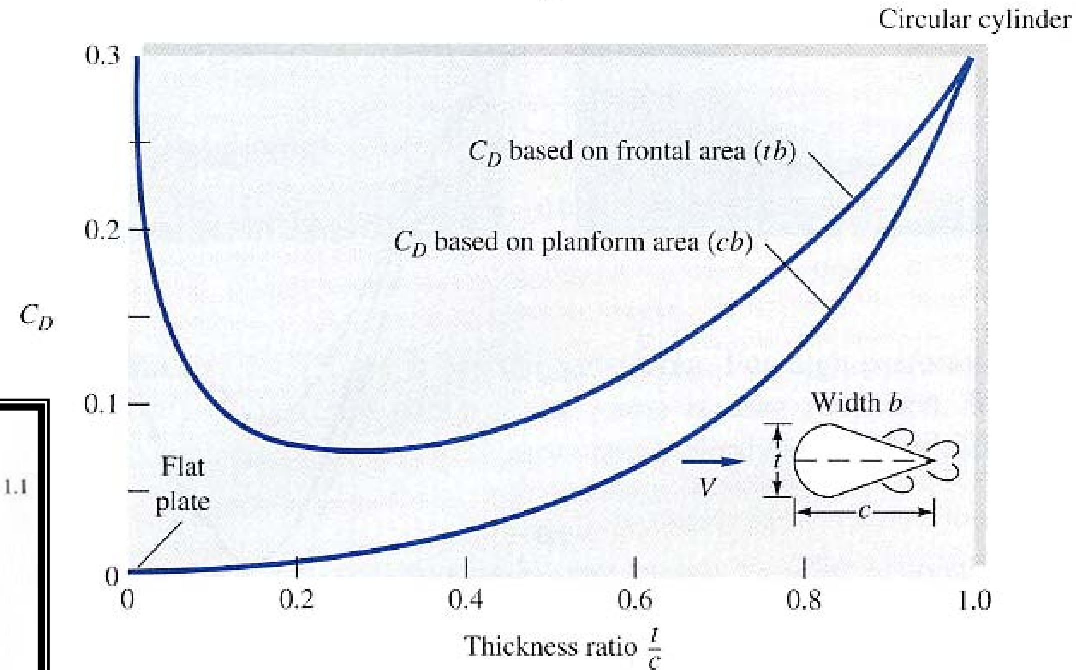
$$F_D = \frac{1}{2}\rho C_D A U^2$$

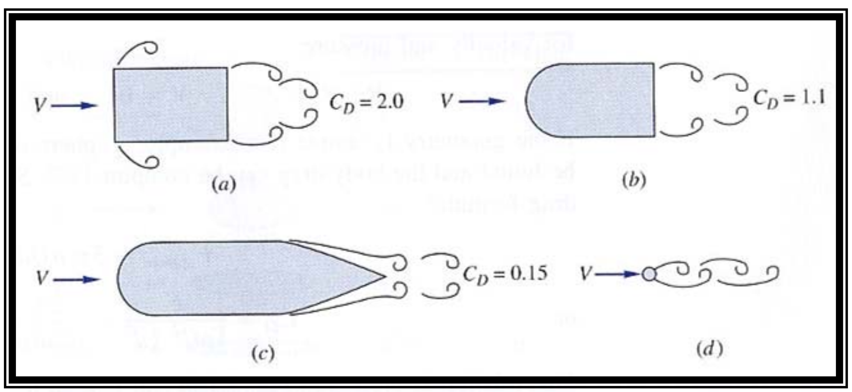
- Where C_D is found empirically through experimentation
- A is profile (frontal) area
- C_D is Reynolds number dependent and is quite different in laminar vs. turbulent flows

Trade-off between Friction and Pressure drag

c = Body length inline with flow
t = Body thickness







From F.M. White, Fluid Mechanics, 5th Ed. McGraw Hill 2005. p. 479, 481.

Drag Coefficient: Cylinder

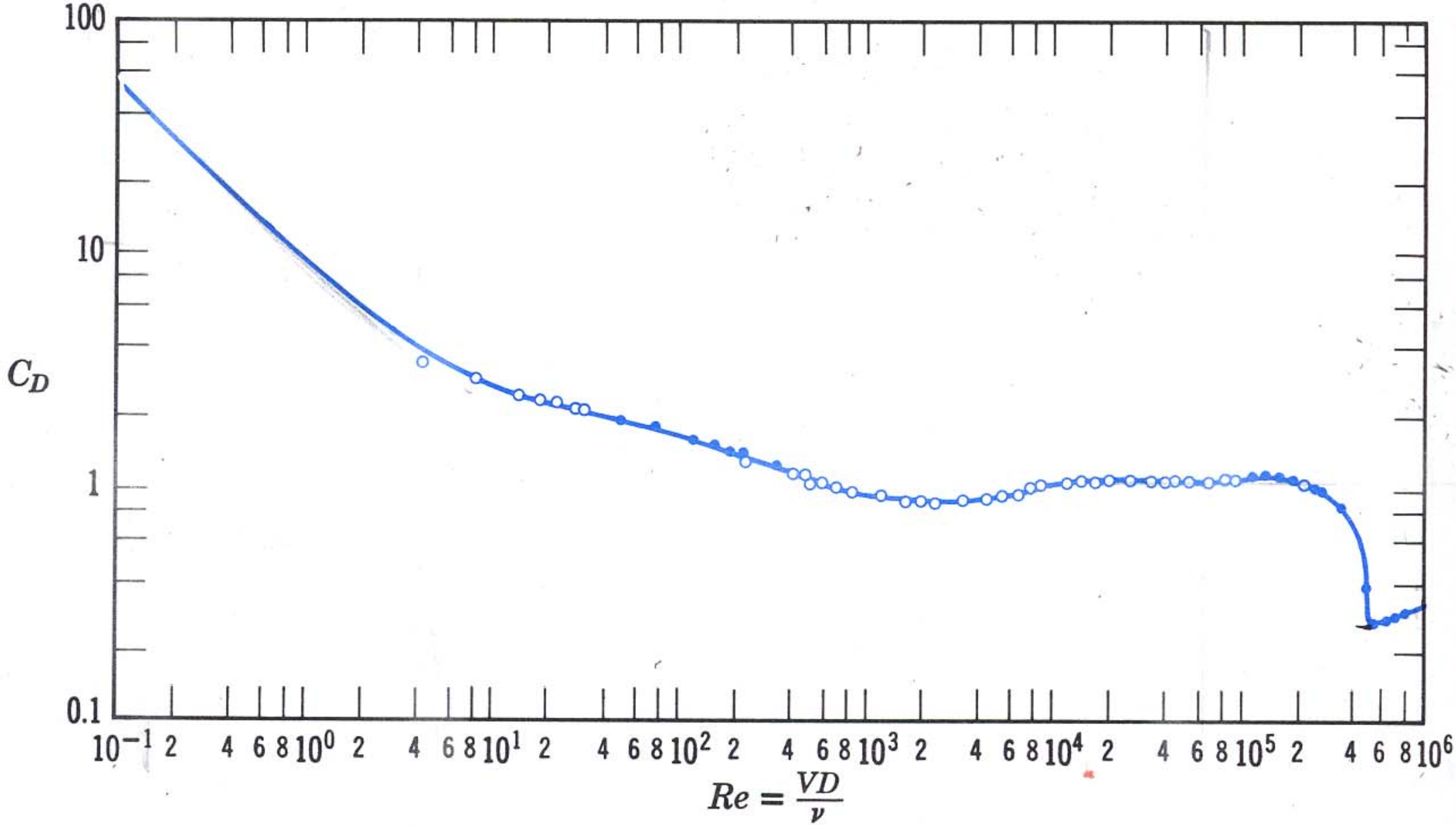


Fig. 9.13 Drag coefficient for a smooth circular cylinder as a function of Reynolds number [3].

Drag Coefficient: Sphere

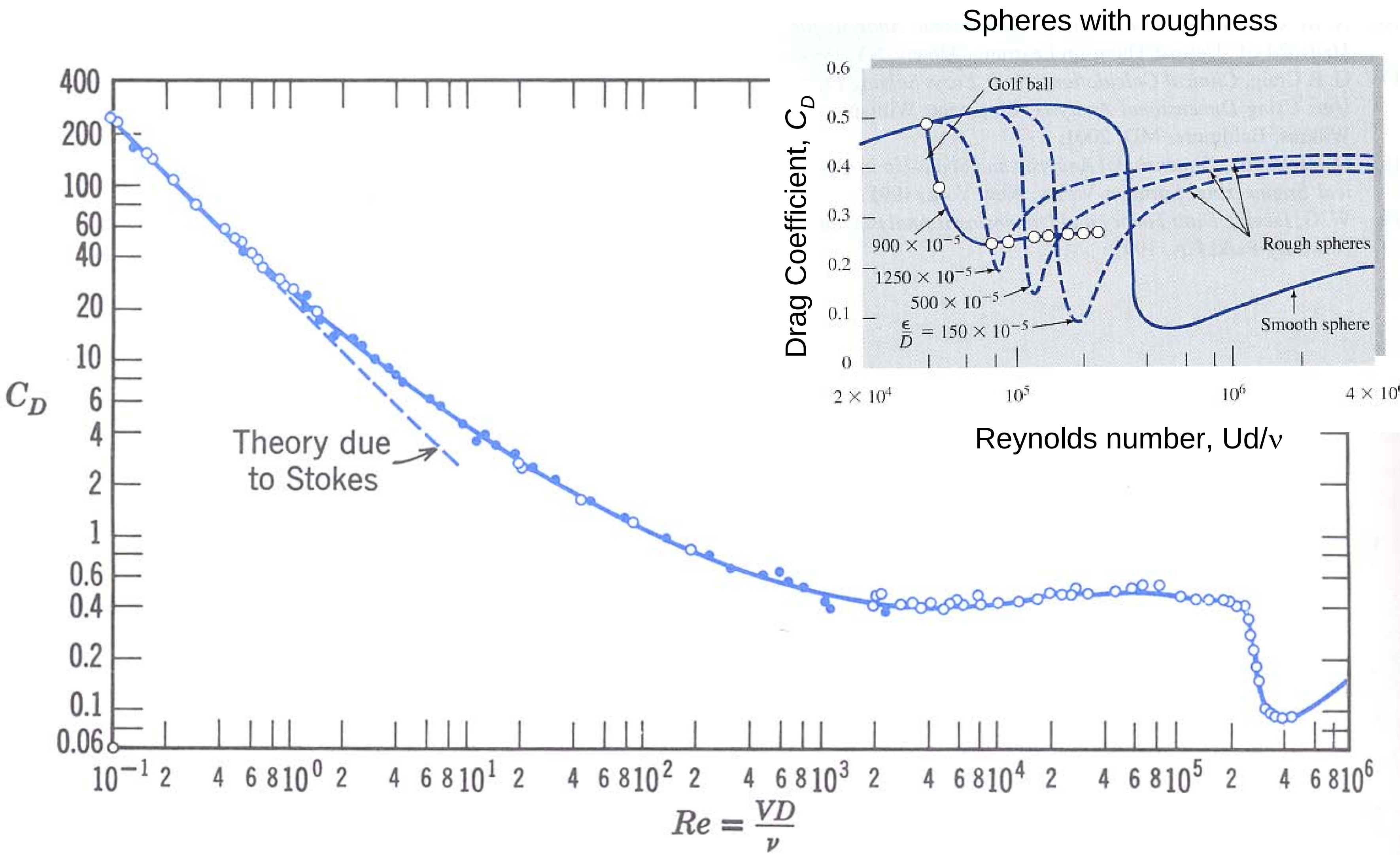
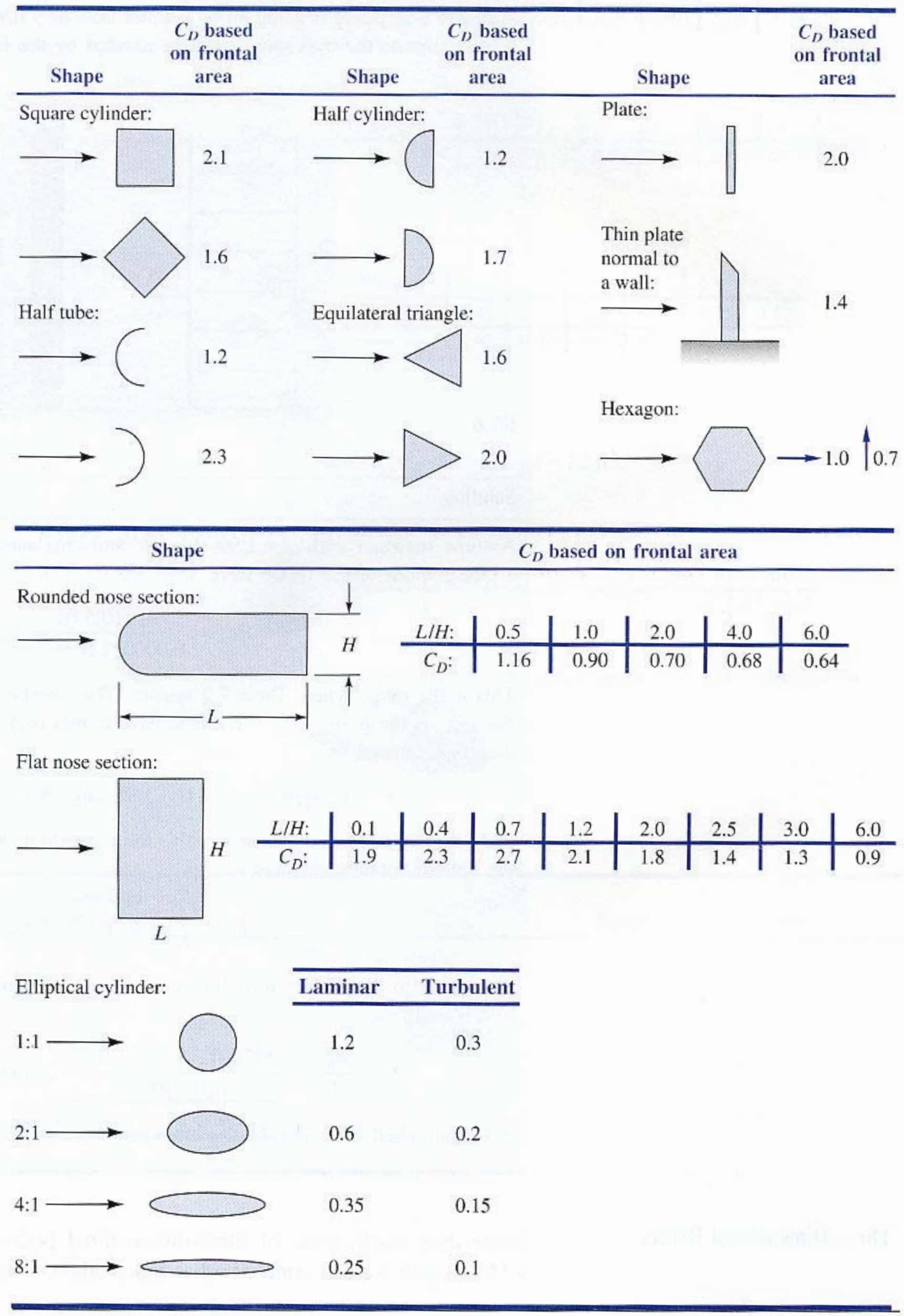


Fig. 9.11 Drag coefficient of a smooth sphere as a function of Reynolds number [3].

Drag Coefficients

For 2D bodies: use C_D to calculate force per unit length.

Use a “strip theory” type approach to determine total drag, assuming that the flow is uniform along the span of the body.



Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder:	2.1	Half cylinder:	1.2	Plate:	2.0
	1.6		1.7	Thin plate normal to a wall:	1.4
Half tube:	1.2	Equilateral triangle:	1.6		
	2.3		2.0	Hexagon:	→1.0 ↑0.7

Shape	C_D based on frontal area
Rounded nose section:	

L/H:	0.5	1.0	2.0	4.0	6.0
C_D:	1.16	0.90	0.70	0.68	0.64

Flat nose section:

L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9

Elliptical cylinder:	Laminar	Turbulent
1:1	1.2	0.3
2:1	0.6	0.2
4:1	0.35	0.15
8:1	0.25	0.1

From F.M. White, Fluid Mechanics, 5th Ed. McGraw Hill 2005. p.483

$C_D \rightarrow$

Table 7.3 Drag of Three-Dimensional Bodies at Re ≥ 10^4

Body	C_D based on frontal area
Cube: (face-on)	1.07
Cube: (edge-on)	0.81
Cup: (concave side facing flow)	1.4
Cup: (convex side facing flow)	0.4
Disk:	1.17
Parachute (Low porosity):	1.2

Streamlined train (approximately 5 cars):

$C_D A \approx 8.5\ m^2$

Bicycle:

Upright: $C_D A \approx 0.51\ m^2$; Racing: $C_D A \approx 0.30\ m^2$

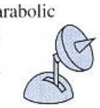
Cone:

θ:	10°	20°	30°	40°	60°	75°	90°
C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

Short cylinder, laminar flow:

L/D:	1	2	3	5	10	20	40	∞
C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

Porous parabolic dish [23]:

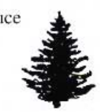


Porosity:	0	0.1	0.2	0.3	0.4	0.5
← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

Average person:

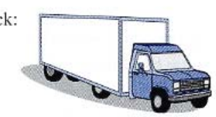
→ $C_D A \approx 9\ ft^2$ ↑ $C_D A \approx 1.2\ ft^2$

Pine and spruce trees [24]:



U, m/s:	10	20	30	40
C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

Tractor-trailer truck:



Without deflector: 0.96; with deflector: 0.76

Body	Ratio	C_D based on frontal area
Rectangular plate:	b/h 1	1.18
	5	1.2
	10	1.3
	20	1.5
	∞	2.0

Body	Ratio	Laminar	Turbulent
Ellipsoid:	L/d 0.75	0.5	0.2
	1	0.47	0.2
	2	0.27	0.13
	4	0.25	0.1
	8	0.2	0.08

Body	Ratio	C_D based on frontal area
Flat-faced cylinder:	L/d 0.5	1.15
	1	0.90
	2	0.85
	4	0.87
	8	0.99
Buoyant rising sphere [50], $135 < Re_d < 1E5$		$C_D \approx 0.95$

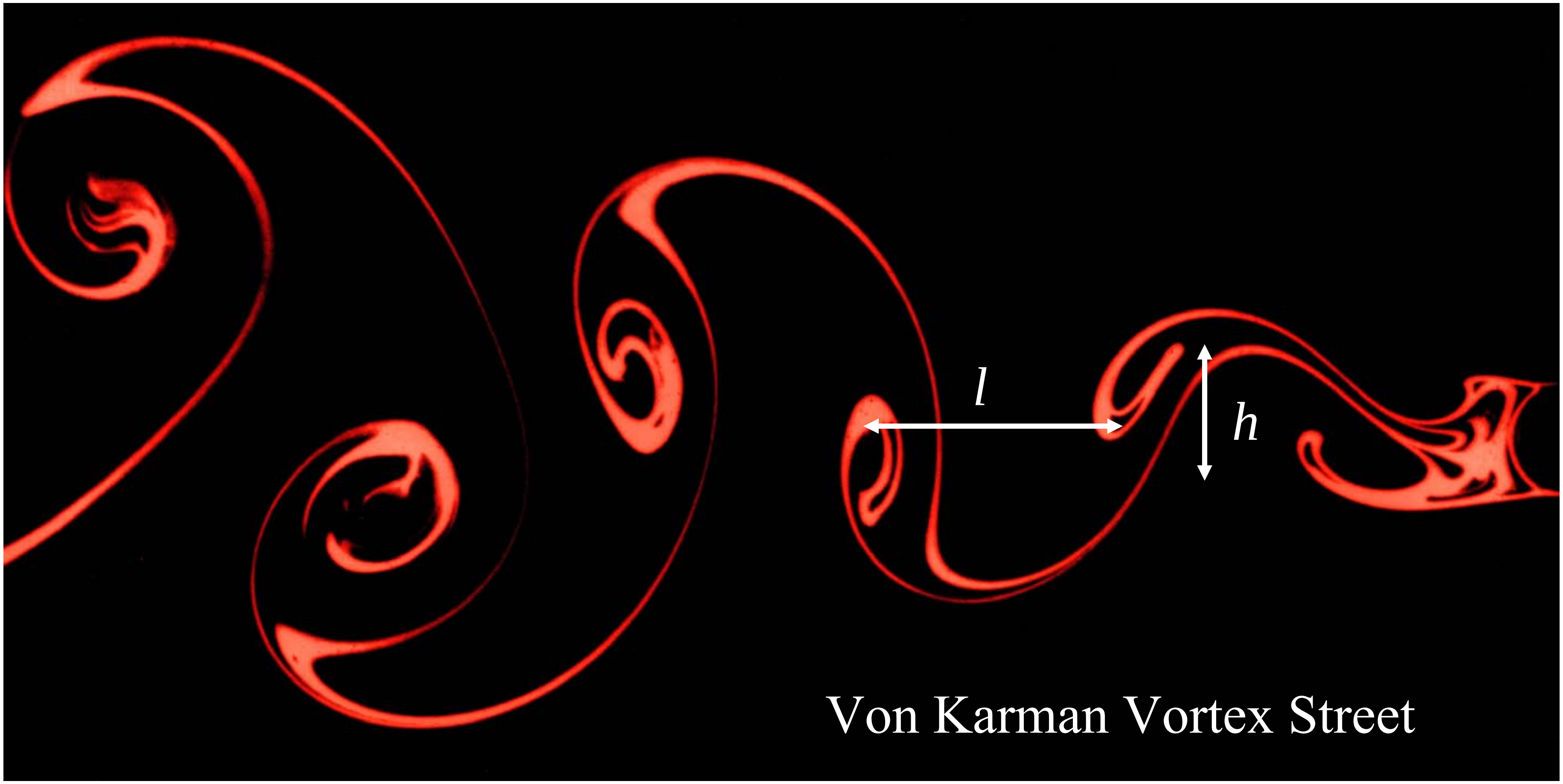
From F.M. White, Fluid Mechanics, 5th Ed. McGraw Hill 2005. p.485

Vortex Shedding

- When the flow separates vortices are shed in the wake.
- Vortices are small “eddies” that result in a force on the body.
- Recall vorticity is defined as the curl of the velocity field:

$$\vec{\omega} = \nabla \times \vec{V}$$

Classical Vortex Shedding



Alternately shed opposite signed vortices

Vortex shedding results from a wake instability

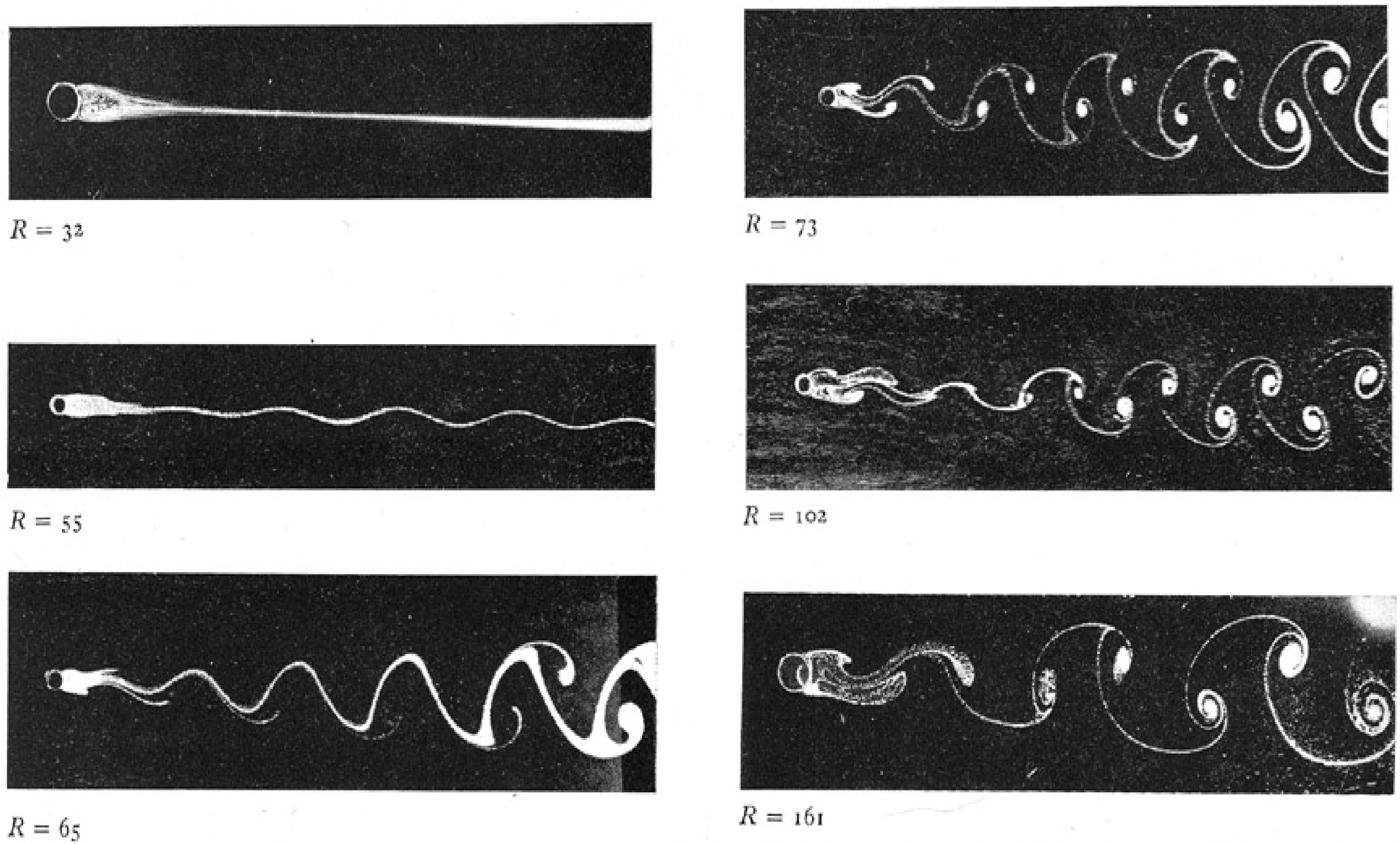


Figure 4.12.6. Streak lines in the wake behind a circular cylinder in a stream of oil. (From Homann 1936*a*.)

Reynolds Number Dependence

$$R_d = \frac{Ud}{\nu} \approx \frac{\text{inertial effects}}{\text{viscous effects}}$$

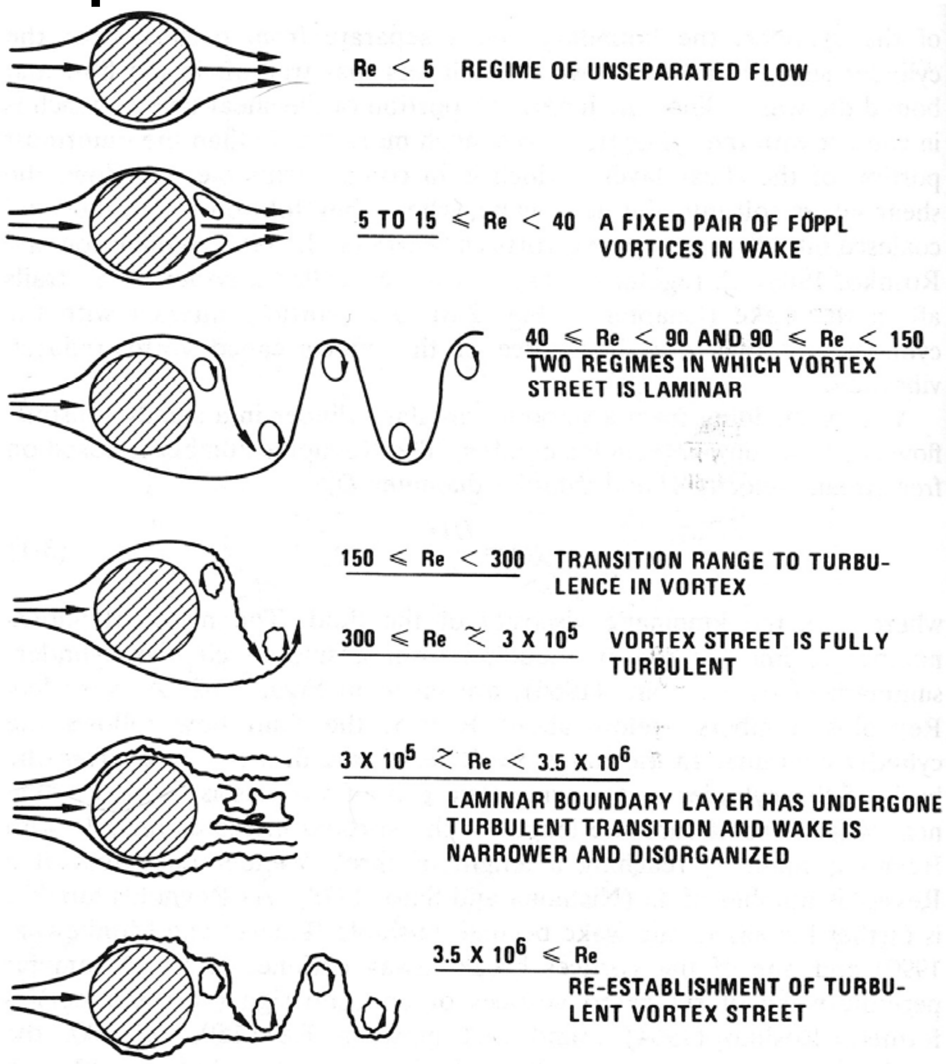


Fig. 3-2 Regimes of fluid flow across smooth circular cylinders (Lienhard, 1966).

$R_d < 5$

$5\text{-}15 < R_d < 40$

$40 < R_d < 150$

$150 < R_d < 300$

Transition to turbulence

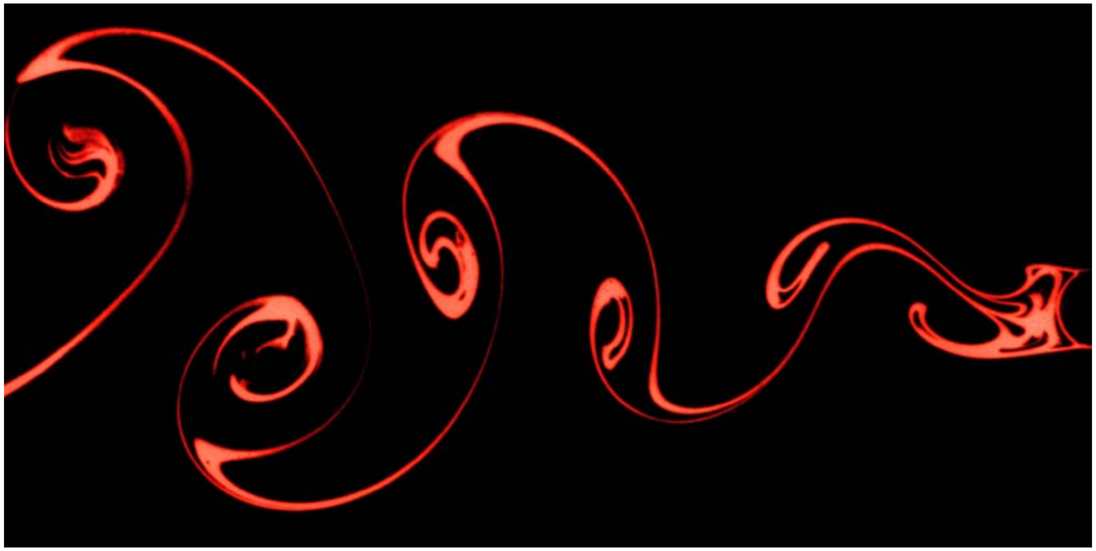
$300 < R_d < 3*10^5$

$3*10^5 < R_d < 3.5*10^6$

$3.5*10^6 < R_d$

From Blevins

Vortex shedding dictated by the Strouhal number



$$S_t = f_s d/U$$

f_s is the shedding frequency, d is diameter and U inflow speed

Strouhal Number vs. Reynolds Number

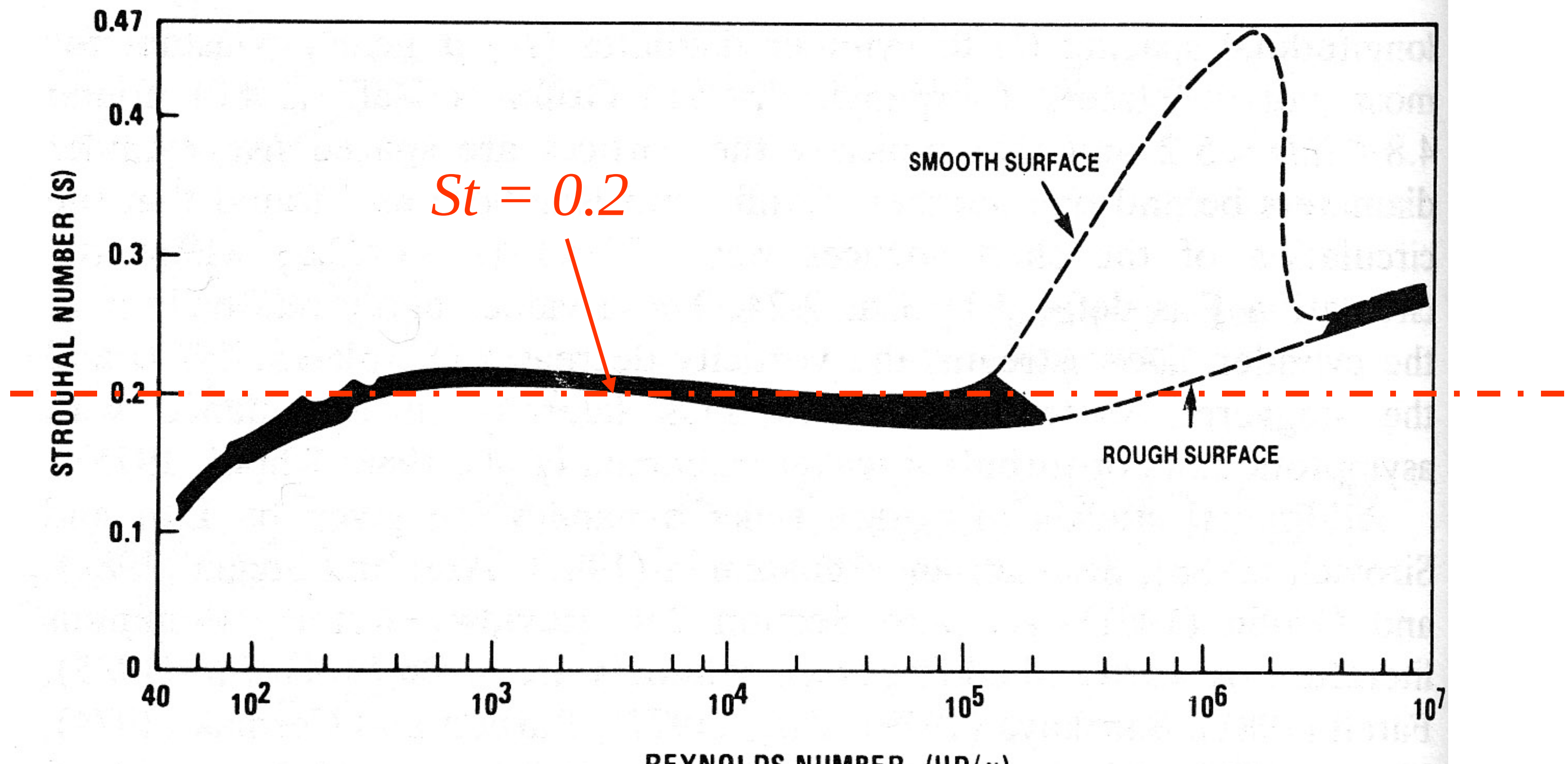


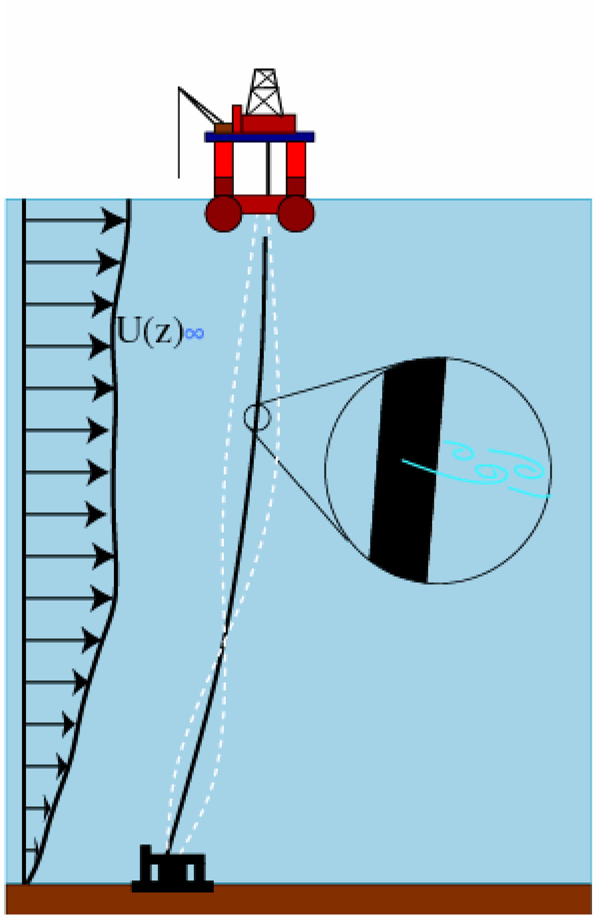
Fig. 3-3 Strouhal number–Reynolds number relationship for circular cylinders (Lienhard, 1966; Achenbach and Heinecke, 1981). $S \approx 0.21\ (1-21/\mathrm{Re})$ for $40 < \mathrm{Re} < 200$ (Roshko, 1955).

Vortex Induced Vibrations

- Vortices shed from bluff bodies are typically shed alternately from top and bottom.
- Each time a vortex is shed there is a resultant force on the body.
- Asymmetrical vortex shedding results in an oscillating force acting on the body transverse (and inline) to the flow.
- This forcing often results in vibrations on long risers and cables.
- These vibrations are called *vortex induced vibrations (VIV)*

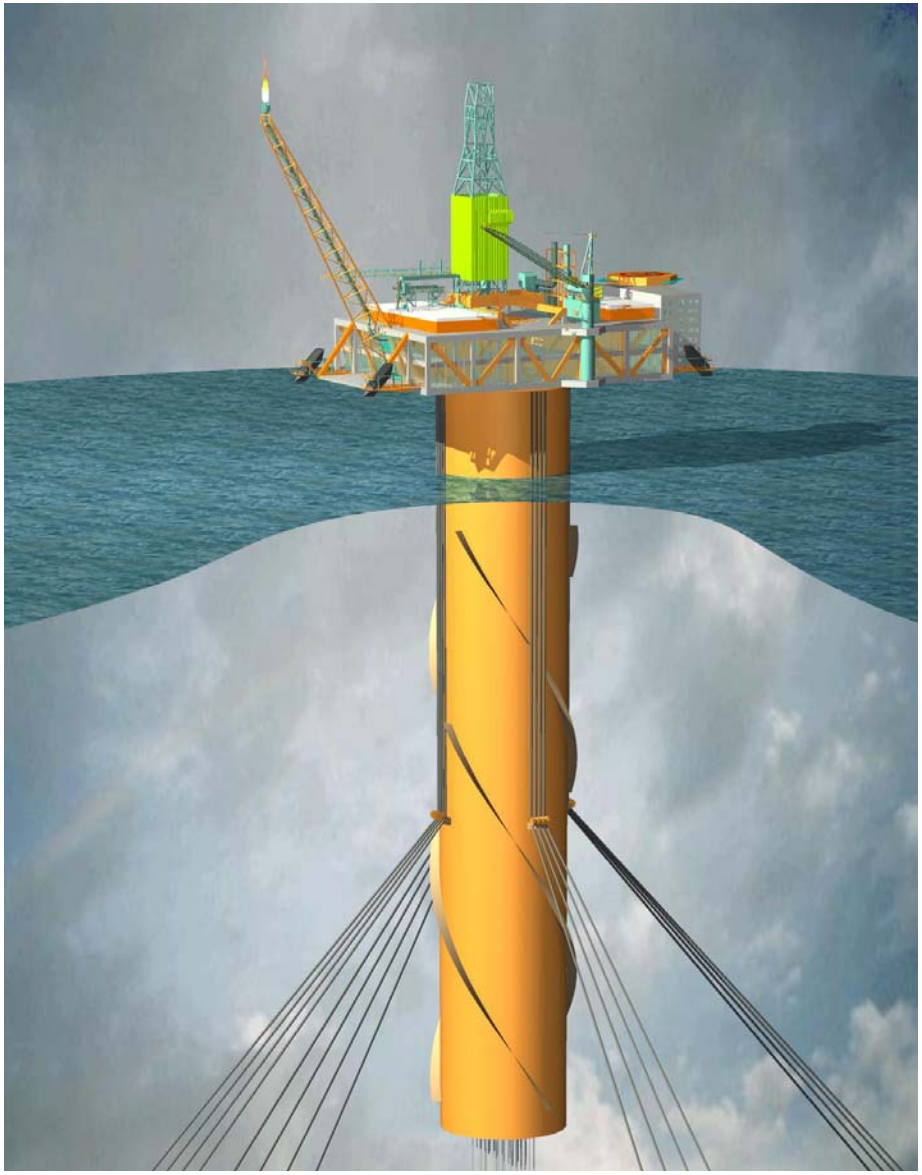
Why is VIV important in Ocean Engineering?

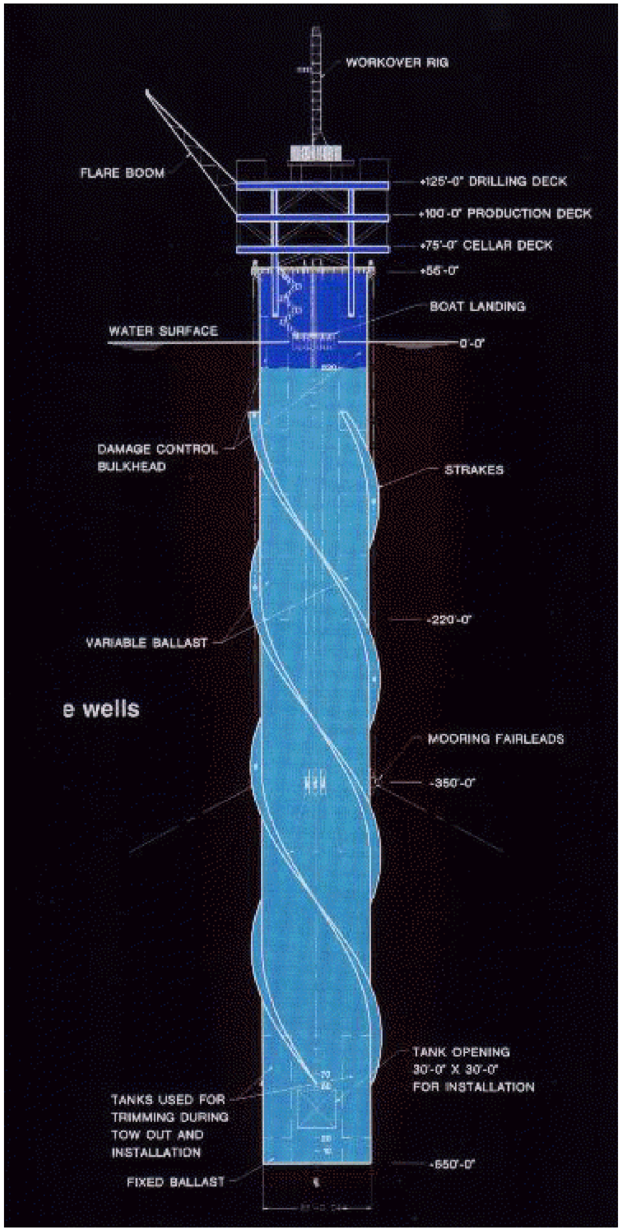
VIV in the Ocean



- Non-uniform currents effect the spanwise vortex shedding on a cable or riser.
- The frequency of shedding can be different along length.
- Understanding the forces on long risers/cables is very challenging despite the simple geometry.

Spar Platforms

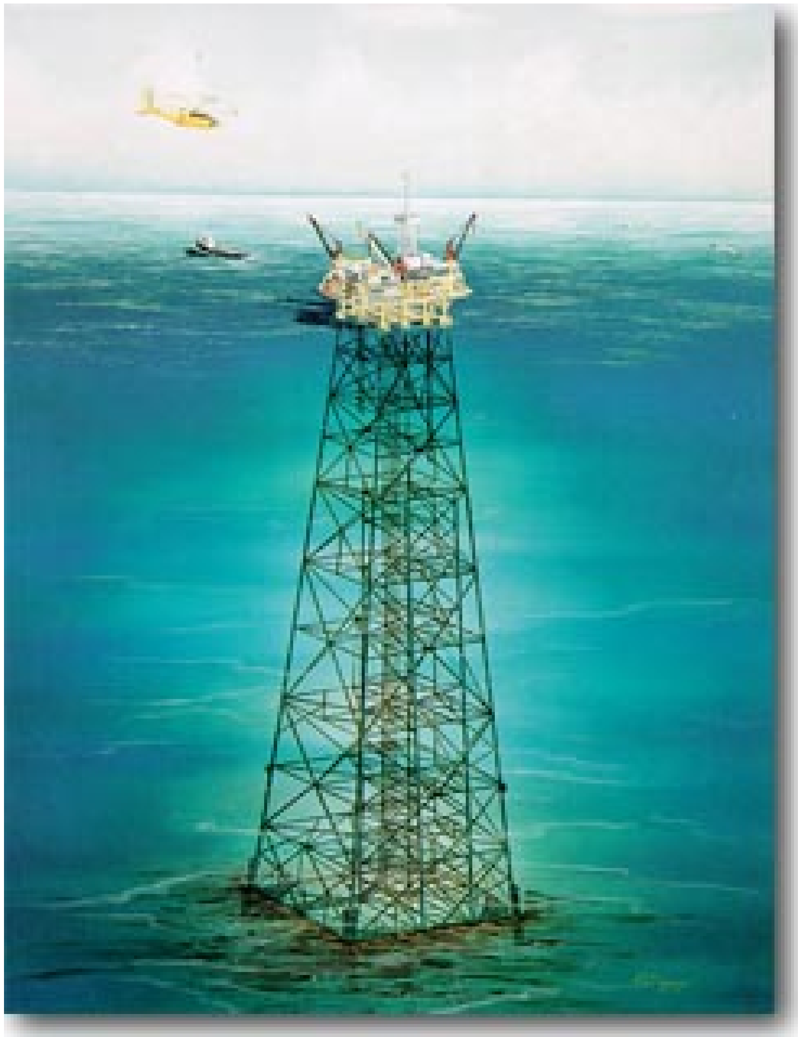




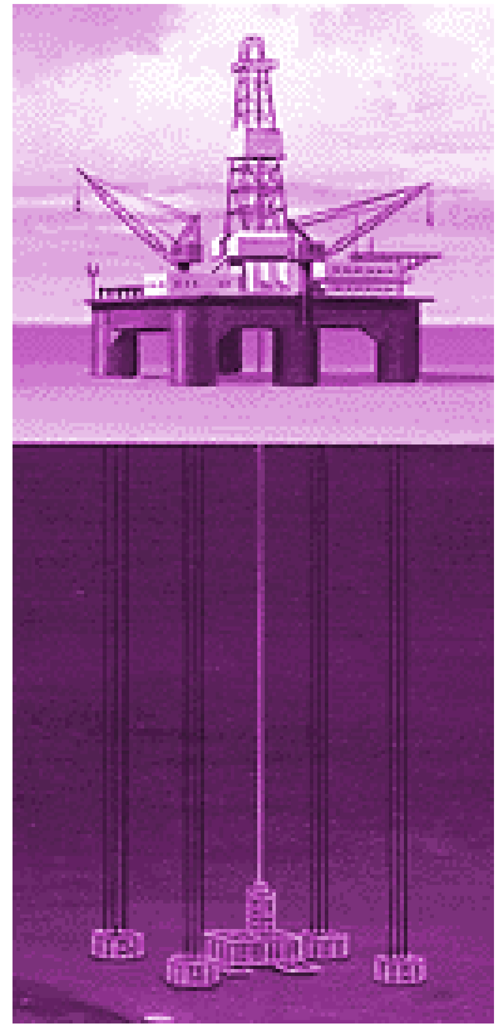
Offshore Platforms

Semi Submersible

Fixed Rigs

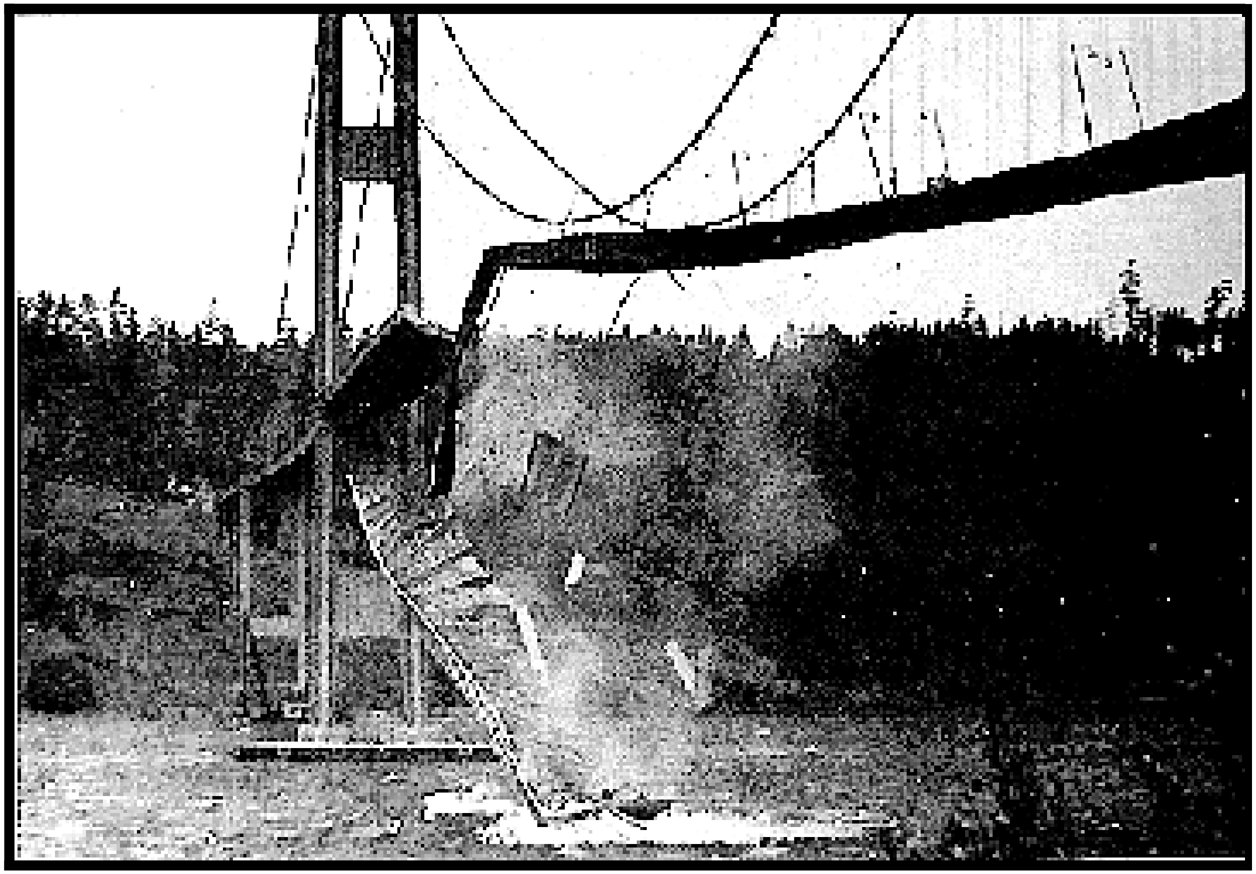


Tension Leg Platforms



Genesis Spar Platform

VIV Catastrophe



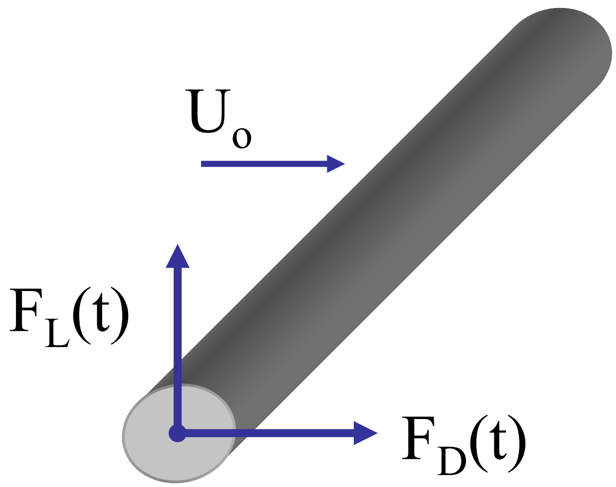
If neglected in design, vortex induced vibrations can prove catastrophic to structures, as they did in the case of the Tacoma Narrows Bridge in 1940.

John Hancock Building

Reprinted from
http://www.glasssteelandstone.com/BuildingDetail/399.php

"**I** n another city, the John Hancock tower wouldn't be anything special -- just another reflective glass box in the crowd. But because of the way Boston and the rest of New England has grown up architecturally, this "70's modern" building stands out from the rest. Instead of being colonial, it breaks new ground. Instead of being quaint, it soars and imposes itself on the skyline. And Instead of being white like so many buildings in the region, this one defies the local conventional wisdom and goes for black. For these reasons and more the people of Boston have fallen in love with the 790-foot monster looming as the tallest building in New England at the time of its completion. In the mid-1990's, The Boston Globe polled local architects who rated it the city's third best architectural structure. Much like Boston's well-loved baseball team, the building has had a rough past, but still perseveres, coming back stronger to win the hearts of its fans. The trouble began early on. During construction of the foundation the sides of the pit collapsed, nearly sucking Trinity Church into the hole. Then in late January, 1973 construction was still underway when a winter storm rolled into town and a 500-pound window leapt from the tower and smashed itself to bits on the ground below. Another followed. Then another. Within a few weeks, more than 65 of the building's 10,344 panes of glass committed suicide, their crystalline essence piling up in a roped-off area surrounding the building. The people of Bean Town have always been willing to kick a brother when he's down, and started calling the tower the Plywood Palace because of the black-painted pieces of wood covering more than an acre of its façade. Some people thought the building was swaying too much in the wind, and causing the windows to pop out. Some thought the foundation had shifted and it was putting stress of the structural geometry. It turns out the culprit was nothing more than the lead solder running along the window frame. It was too stiff to deal with the kind of vibrations that happen every day in thousands of office buildings around the world. So when John Hancock Tower swayed with the wind, or sighed with the temperature, the windows didn't and eventually cracked and plummeted to Earth. It cost $7,000,000.00 to replace all of those panes of glass. The good news is, you can own a genuine piece of the skyscraper. According to the Globe, the undamaged sheets were sold off for use as tabletops, so start combing those garage sales. For any other skyscraper, the hardship would end there. But the Hancock building continued to suffer indignities. The last, and most ominous, was revealed by Bruno Thurlimann, a Swiss engineer who determined that the building's natural sway period was dangerously close to the period of its torsion. The result was that instead of swaying back-and-forth like a in the wind like a metronome, it bent in the middle, like a cobra. The solution was putting a pair of 300-ton tuned mass dampeners on the 58-th floor. The same engineer also determined that while the $3,000,000.00 mass dampeners would keep the building from twisting itself apart, the force of the wind could still knock it over. So 1,500 tons of steel braces were used to stiffen the tower and the Hancock building's final architectural indignity was surmounted."

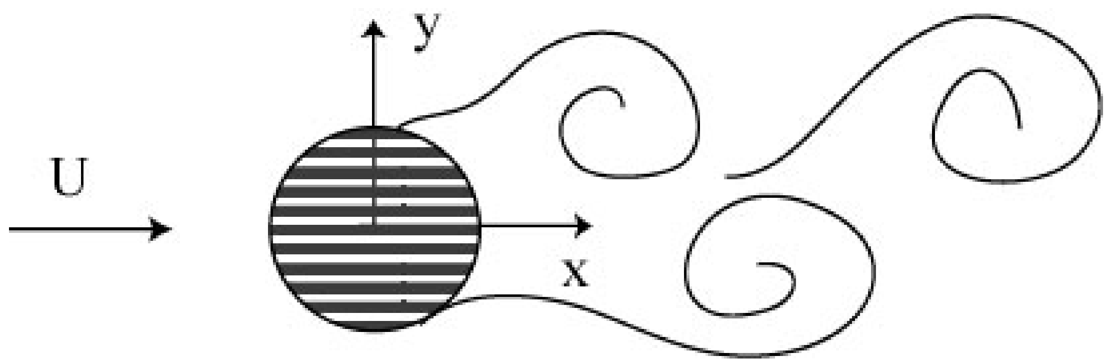
Vortex Shedding Generates forces on Cylinder



Both Lift and Drag forces persist on a cylinder in cross flow. Lift is perpendicular to the inflow velocity and drag is parallel.

Due to the alternating vortex wake ("Karman street") the oscillations in lift force occur at the vortex shedding frequency and oscillations in drag force occur at *twice* the vortex shedding frequency.

Vortex Induced Forces

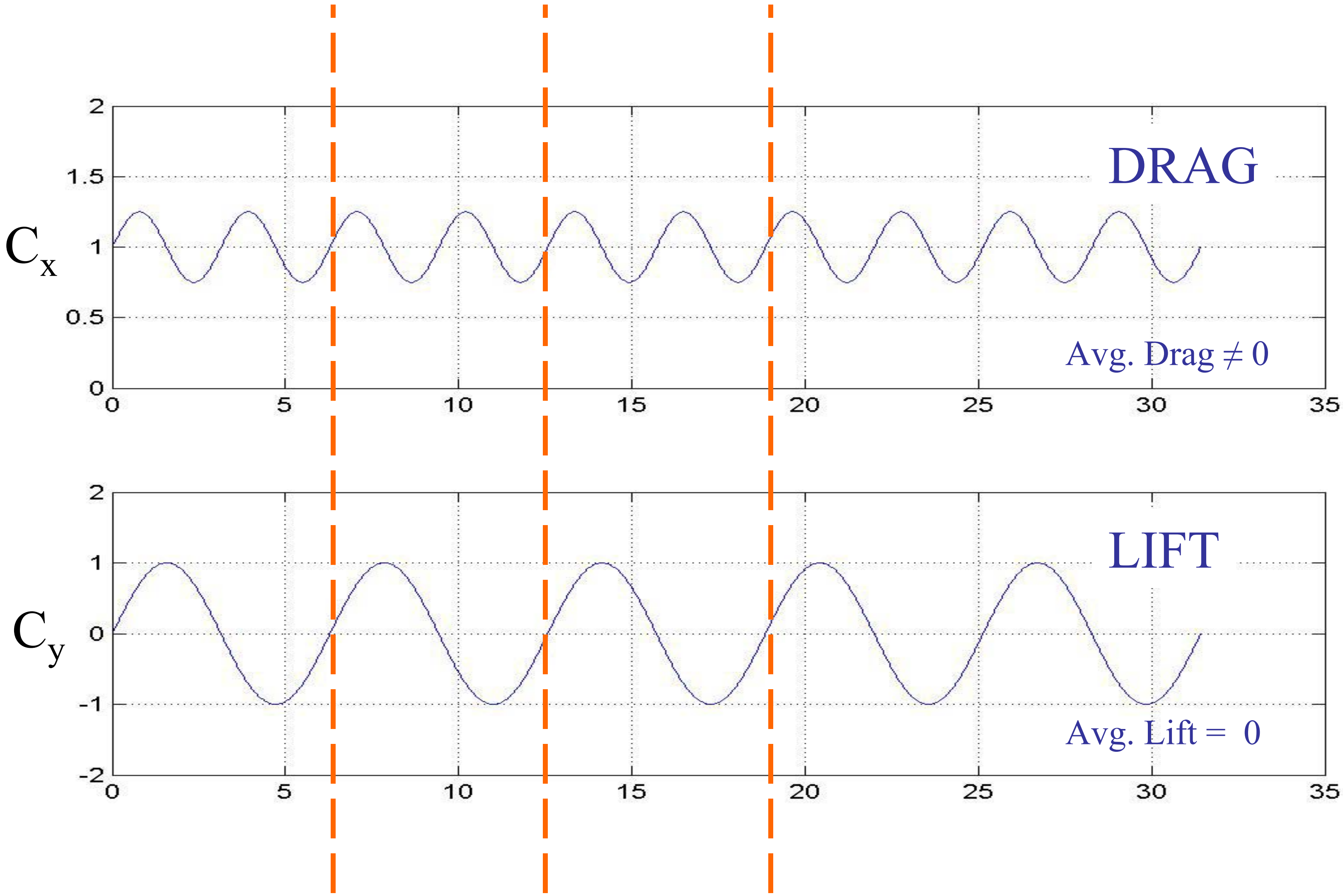


Due to unsteady flow, forces, X(t) and Y(t), vary with time.

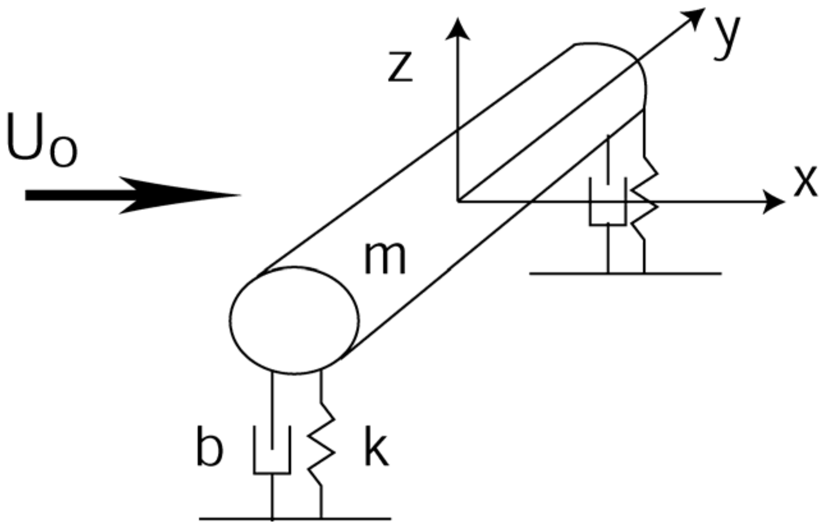
Force coefficients:

$$C_x = \frac{D(t)}{{}^1/_2\, \rho\, U^2\, d} \qquad C_y = \frac{L(t)}{{}^1/_2\, \rho\, U^2\, d}$$

Force Time Trace



Alternate Vortex shedding causes oscillatory forces which induce structural vibrations



Rigid cylinder is now similar to a spring-mass system with a harmonic forcing term.

Heave Motion z(t)

$$z(t) = z_o \cos \omega t$$

$$\dot{z}(t) = -z_o \omega \sin \omega t$$

$$\ddot{z}(t) = -z_o \omega^2 \cos \omega t$$

LIFT = $L(t) = Lo \cos(\omega_s t + \psi)$

DRAG = $D(t) = Do \cos(2\omega_s t + \psi)$

$\omega_s = 2\pi f_s$

“Lock-in”

A cylinder is said to be “locked in” when the frequency of oscillation is equal to the frequency of vortex shedding. In this region the largest amplitude oscillations occur.

Shedding frequency

$$\omega_v = 2\pi f_v = 2\pi S_t (U/d)$$

Natural frequency of oscillation

$$\omega_n = \sqrt{\frac{k}{m + m_a}}$$

Equation of Cylinder Heave due to Vortex shedding

$$m\ddot{z} + b\dot{z} + kz = L(t)$$

$$L(t) = -L_a\,\ddot{z}(t) + L_v\,\dot{z}(t)$$

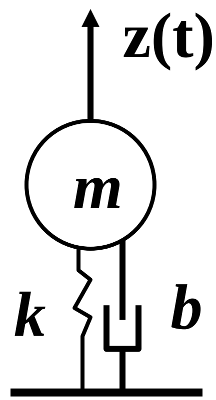
$$m\ddot{z}(t) + b\dot{z}(t) + kz(t) = -L_a\,\ddot{z}(t) + L_v\,\dot{z}(t)$$

$$\underbrace{(m + L_a)\ddot{z}(t)} + \underbrace{(b - L_v)\dot{z}(t)} + \underbrace{k\,z(t)} = 0$$

Added mass term

Damping → *If $L_v > b$ system is UNSTABLE*

Restoring force



Lift Force on a Cylinder

Lift force is sinusoidal component and residual force. Filtering the recorded lift data will give the sinusoidal term which can be subtracted from the total force.

LIFT FORCE: $L(t) = L_o \cos(\omega t + \psi_o) \quad if \;\; \omega < \omega_v$

$$L(t) = L_o \cos \omega t \cos \psi_o - L_o \sin \omega t \sin \psi_o$$

$$L(t) = \frac{-L_o \cos \psi_o}{z_o \omega^2} \ddot{z}(t) + \frac{L_o \sin \psi_o}{z_o \omega} \dot{z}(t)$$

where ω_v is the frequency of vortex shedding

Lift Force Components:

Two components of lift can be analyzed:

Lift in phase with acceleration (added mass):

$$M_a(\omega, a) = \frac{L_o}{a\omega^2}\cos\psi_o$$

Lift in-phase with velocity:

$$L_v = -\frac{L_o}{a\omega}\sin\psi_o$$

Total lift:

$$L(t) = -M_a(\omega, a)\,\ddot{z}(t) + L_v(\omega, a)\,\dot{z}(t)$$

(a = z_o is cylinder heave amplitude)

Total Force:

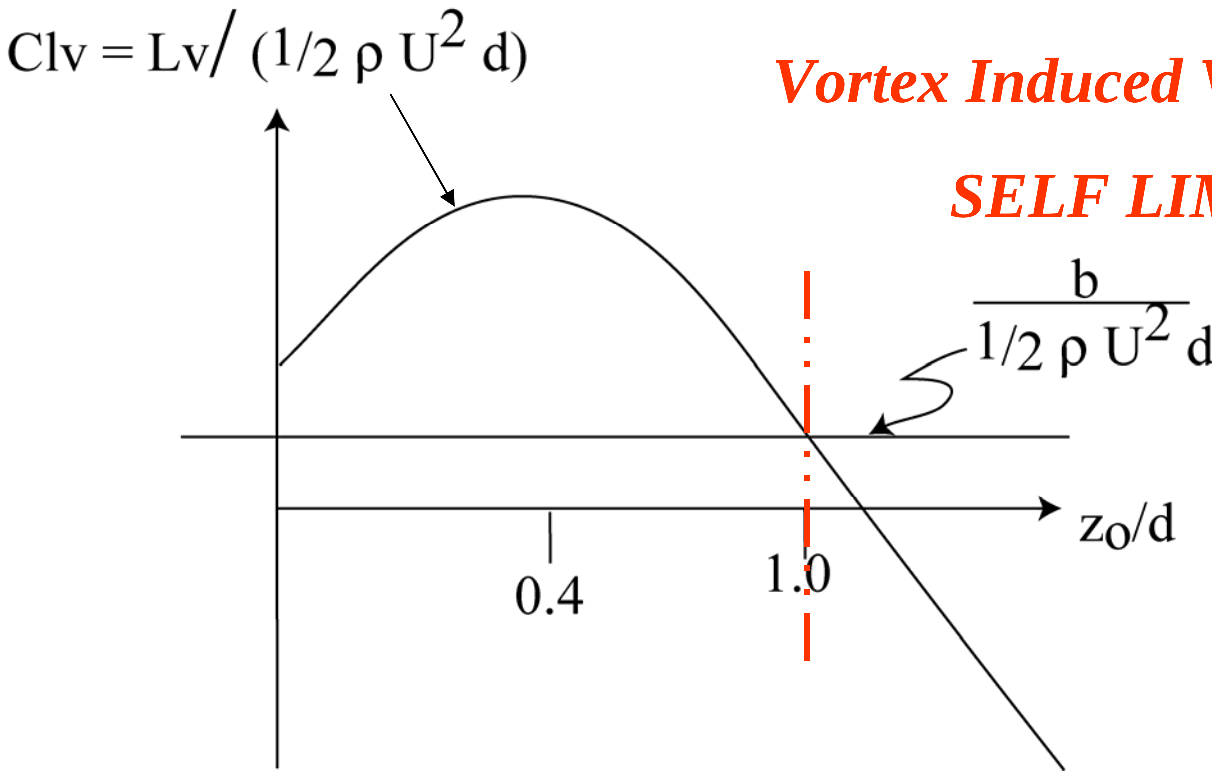
$$L(t) = -M_a(\omega, a)\,\ddot{z}(t) + L_v(\omega, a)\,\dot{z}(t)$$

$$= -\left(\tfrac{\pi}{4}\rho d^2\right) C_{ma}(\omega, a)\,\ddot{z}(t)$$

$$+\left(\tfrac{1}{2}\rho d U^2\right) C_{Lv}(\omega, a)\,\dot{z}(t)$$

- If $\boldsymbol{C_{Lv} > 0}$ then the fluid force amplifies the motion instead of opposing it. This is self-excited oscillation.
- C_{ma}, C_{Lv} are dependent on ω and a.

Coefficient of Lift in Phase with Velocity



Vortex Induced Vibrations are

SELF LIMITED

In air: ρ_{air} ~ small, z_{max} ~ 0.2 diameter

In water: ρ_{water} ~ large, z_{max} ~ 1 diameter

Amplitude Estimation

Blevins (1990)

$$a/_{d} \simeq 1.29/[1+0.43\, S_G]^{3.35}$$

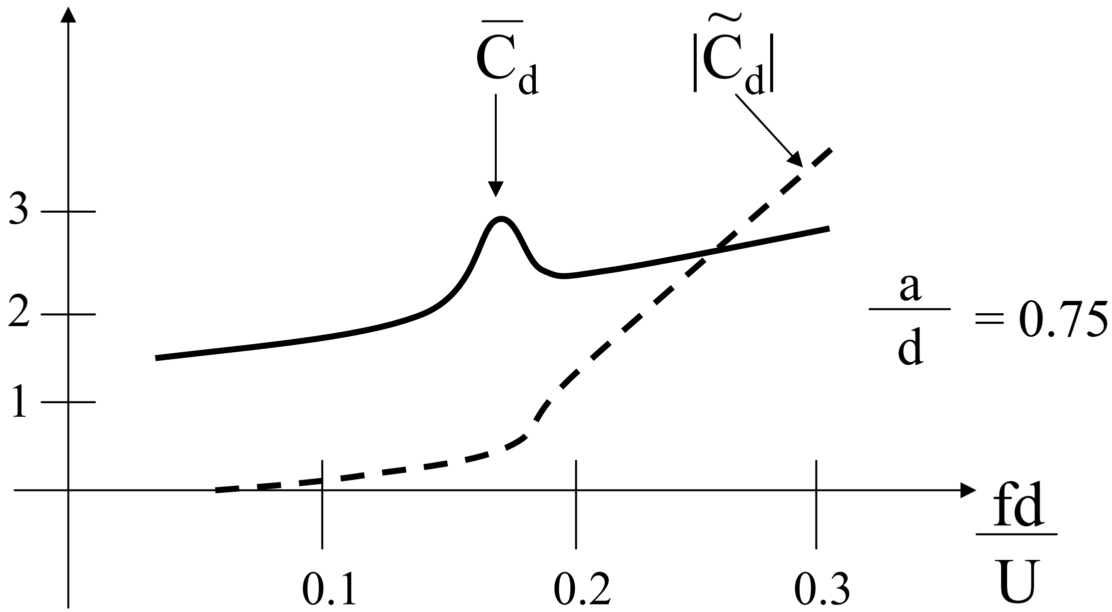
$$S_G = 2\,\pi\,\hat{f}_n^{\,2}\,\frac{2\bar{m}\,(2\pi\zeta)}{\rho\, d^2}\;;\;\hat{f}_n = f_n/f_s;\quad \bar{m} = m + m_a^*$$

$$\zeta = \frac{b}{2\sqrt{k(m+m_a^*)}}$$

$m_a^* = \rho\ \forall\, C_{ma}$; where $C_{ma} = 1.0$

Drag Amplification

VIV tends to increase the effective drag coefficient. This increase has been investigated experimentally.



Gopalkrishnan (1993)

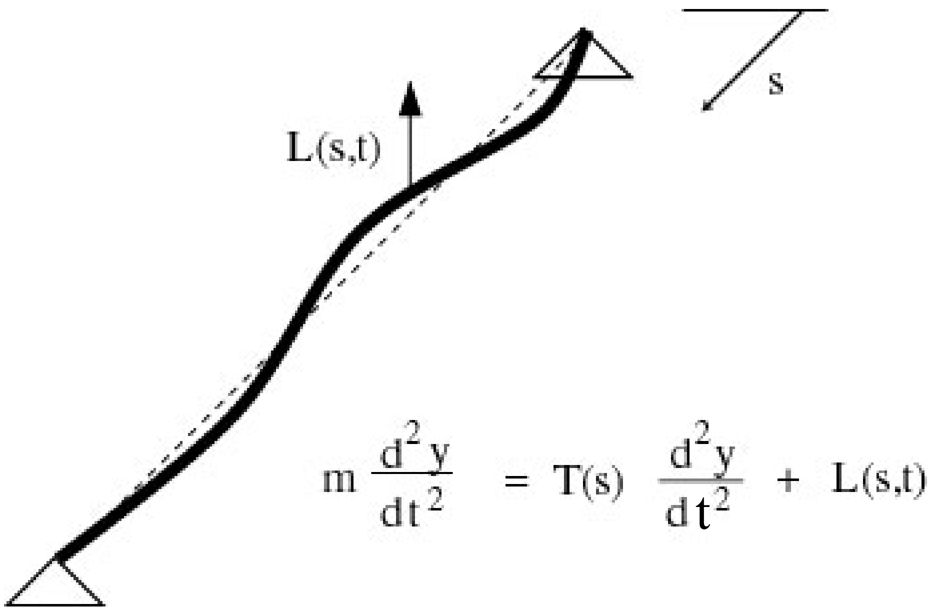
Mean drag:

$$\overline{C}_d = 1.2 + 1.1\left(a/_d\right)$$

Fluctuating Drag:

$\tilde{C}_d$ occurs at twice the shedding frequency.

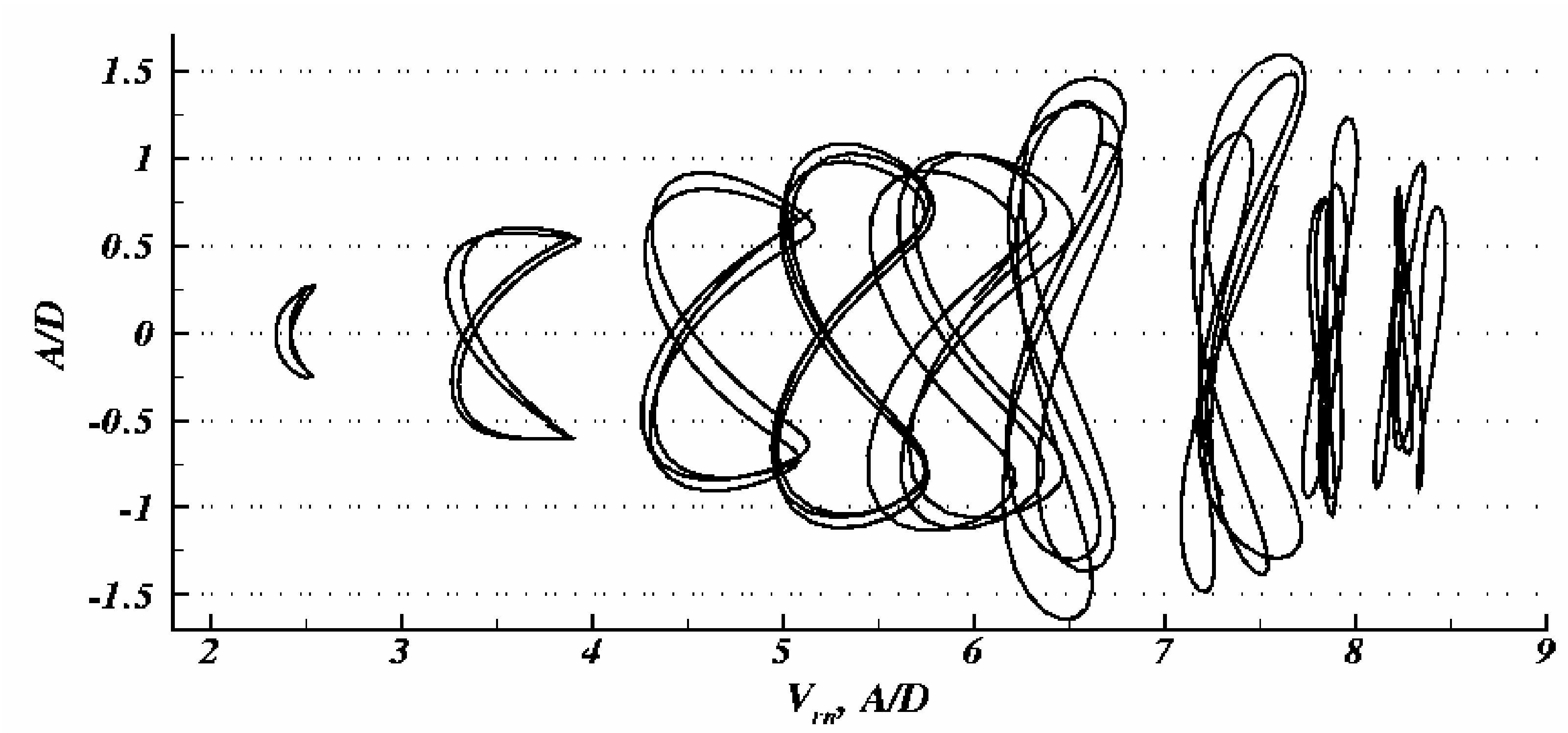
Flexible Cylinders



Mooring lines and towing cables act in similar fashion to rigid cylinders except that their motion is not spanwise uniform.

Tension in the cable must be considered when determining equations of motion

Flexible Cylinder Motion Trajectories



Long flexible cylinders can move in two directions and tend to trace a figure-8 motion. The motion is dictated by the tension in the cable and the speed of towing.

VIV Suppression

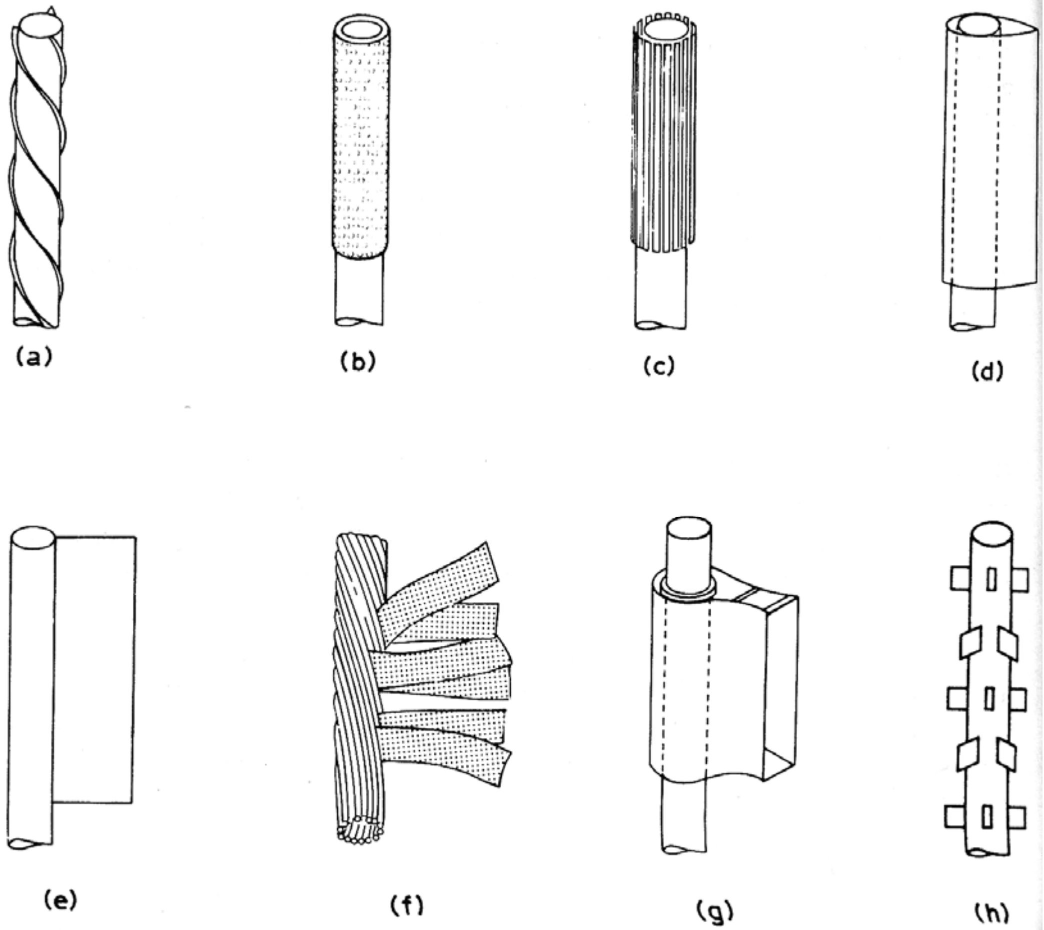
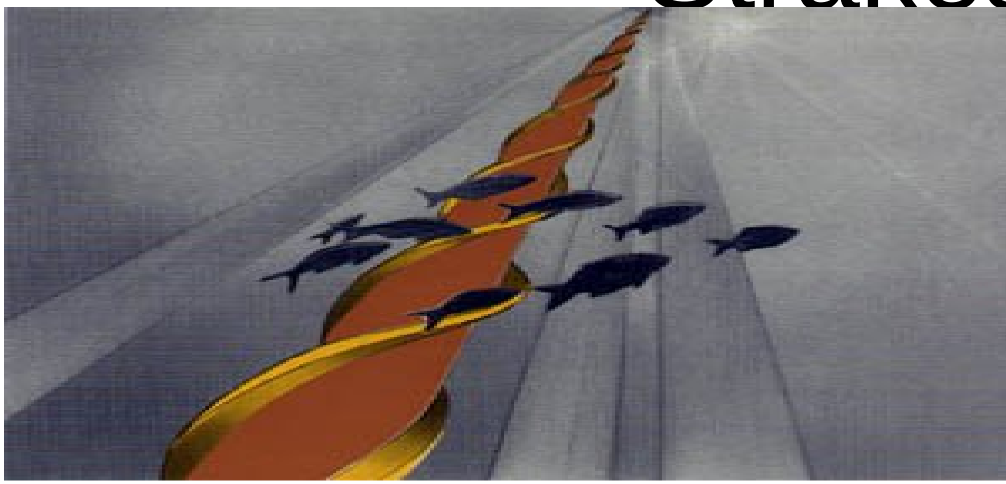


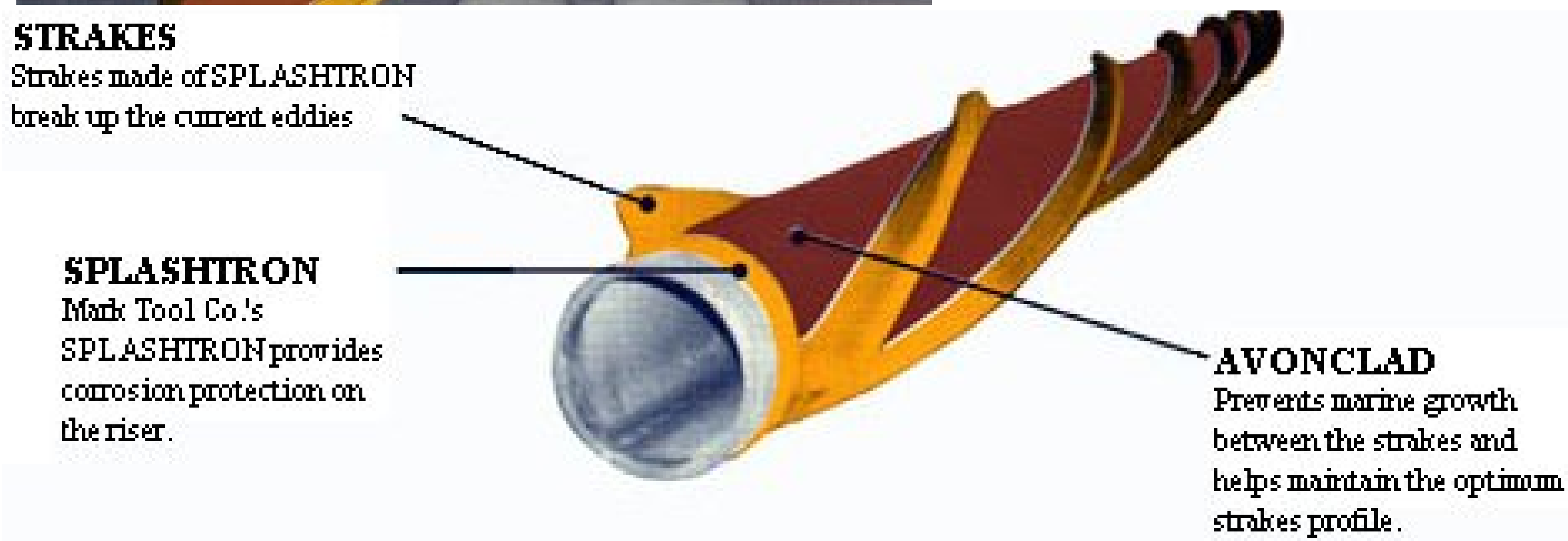
Fig. 3-23 Add-on devices for suppression of vortex-induced vibration of cylinders: (a) helical strake; (b) shroud; (c) axial slats; (d) streamlined fairing; (e) splitter; (f) ribboned cable; (g) pivoted guiding vane; (h) spoiler plates.

- Helical strake
- Shroud
- Axial slats
- Streamlined fairing
- Splitter plate
- Ribboned cable
- Pivoted guiding vane
- Spoiler plates

VIV Suppression by Helical Strakes



Helical strakes are a common VIV suppresion device.



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- White, F.M. (2005) **Fluid Mechanics, 5th Ed.** McGraw Hill, New York.