

Object Impact on the Free Surface and Added Mass Effect

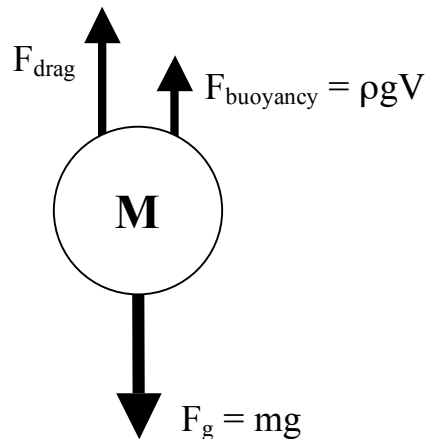
2.016 Laboratory Fall 2005

Prof. A. Techet

Introduction to Free Surface Impact

Free surface impact of objects has applications to ocean engineering such as ship slamming hydrodynamics. The simplest geometric object to study is a sphere. The hydrodynamics are three dimensional, but several basic concepts can be observed using high speed video sequences.

The main focus of Part A of this laboratory exercise is to determine the *terminal velocity* of a sphere that impacts the free surface of a tank of water at high speeds. When an object which is falling under the influence of gravity or subject to some other constant driving force is subject to a resistance (drag force) which increases with velocity, it will ultimately reach a maximum velocity where the drag force equals the driving force. This final, constant velocity of motion is called a "terminal velocity", a terminology made popular by skydivers. For objects moving through a fluid at low speeds so that turbulence is not a major factor, the terminal velocity is determined by viscous drag.



$$F_{drag} + F_{buoyancy} = F_{gravity} \quad (1.1)$$

The drag force is

$$F_{drag} = \frac{1}{2} \rho U^2 C_d A \quad (1.2)$$

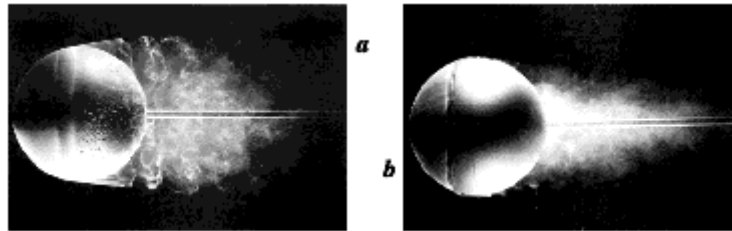
where U is the speed of the object, A is the frontal area of the object, and $C_d = 0.5$ is the coefficient of drag for a sphere.

The buoyancy force is

$$F_{buoyancy} = \rho g \mathcal{V} \quad (1.3)$$

and force of gravity is simply

$$F_{gravity} = mg \quad (1.4)$$



Flow about a sphere in a wind tunnel.

Lab part A:

In this section of the lab you will observe objects impacting the free surface at two speeds and continuing down into the tank. You are asked to qualitatively consider the behavior of the sphere at the moment of impact: the splash formation (height and shape), wave generation, bubble/cavity formation behind the sphere, etc. You will take high speed video of the objects and use a software package to determine the position of the ball as a function of time, $x(t)$. A graduate student will show you how to operate the image recognition software. You will then use the data to determine the velocity of the ball as a function of time.

As the ball approaches the bottom of the tank it may reach terminal velocity. Using the video recorded during the experiments you will determine whether this occurs and how this value compares with the expected value calculated using equation(1.1).

Spinning Sphere: A case of a sphere spinning at a high rate of rotation will be demonstrated and video of the event will be posted on the website. Consider the physics associated with this run. Think about hitting a tennis ball or kicking a soccer ball with top spin. When the ball reaches a certain depth it will make a sharp turn in one direction. What direction do you expect it to turn compared to its direction of spin? Think about the force balance on the sphere towards the end of the run and how the sphere can travel sideways given that gravity is acting downwards on the object.

Write Up:

- For the two non-spinning sphere cases:
 - Briefly describe and discuss the following features of the short moment of impact: the splash shape, height, wave pattern generated, and the cavity formation behind the ball. Discuss how this cavity might affect the speed of the ball as it goes through the water?
 - Determine the terminal velocity analytically.
 - Plot dx/dt from the high-speed camera data, and determine if the ball has reached terminal velocity. Compare dx/dt from the camera to the terminal velocity calculation you made in the pre-lab assignment.
 - How do the two different speed runs compare? Is the terminal velocity the same? Different? Does this make sense in light of the definition of terminal velocity?
- For the one spinning sphere case:
 - Compare the hydrodynamics features (splash shape, height, etc.) at the moment of impact for a spinning and non-spinning sphere.
 - Describe the trajectory of the sphere and the bubble that forms at impact.
 - Discuss the direction that the sphere turns compared to its direction of spin
 - Try to rationalize the behavior of the spinning sphere considering the basic force balance on the sphere towards the end of the run, including how the sphere can travel sideways given that gravity is acting downwards on the object.

Introduction to Added Mass

For the case of unsteady motion of bodies underwater or unsteady flow around objects, we must consider an additional effect (force) acting on the structure when formulating the system equation.

A typical mass-spring-dashpot system can be described by the following equation:

$$m\ddot{x} + b\dot{x} + kx = f(t) \quad (1.5)$$

where m is the system mass, b is the linear damping coefficient, k is the spring coefficient, $f(t)$ is a driving force acting on the mass, and x is the displacement of the mass. The natural frequency ω of the system is simply

$$\omega = \sqrt{\frac{k}{m}}. \quad (1.6)$$

Given an object of mass m attached to a spring, you can determine the spring constant by setting the mass in motion and observing the frequency of oscillation, then using equation (1.6) to solve for k . Alternately we can also determine the spring coefficient k by simply applying a force f and measuring the displacement x :

$$f = kx. \quad (1.7)$$

The apparent mass of an object in air differs from the apparent mass of an object in water. Statically, the buoyancy force acting on the body makes it appear less massive. Using a load (force) cell to measure the weight Mg of an object will reveal this disparity quite clearly. It is important to take this into account when formulating the natural frequency of a spring-mass system in water. In this lab we will compare the natural frequency of such a spring-mass system in air and in water. In addition to the buoyancy effect, an added mass term must be considered.

In a physical sense, this added mass is the weight added to a system due to the fact that an accelerating or decelerating body (ie. unsteady motion: $dU/dt \neq 0$) must move some volume of surrounding fluid with it as it moves. The added mass force opposes the motion and can be factored into the system equation as follows:

$$m\ddot{x} + b\dot{x} + kx = f(t) - m_a\ddot{x} \quad (1.8)$$

where m_a is the added mass. Reordering the terms the system equation becomes:

$$(m + m_a)\ddot{x} + b\dot{x} + kx = f(t) \quad (1.9)$$

From here we can treat this again as a simple spring-mass-dashpot system with a new mass $m' = m + m_a$ such that the natural frequency of the system is now

$$\omega' = \sqrt{\frac{k}{m'}} = \sqrt{\frac{k}{m + m_a}} \quad (1.10)$$

Theoretically the added mass of simple geometric shapes can be calculated using the following formulas:

Sphere:
$$m_a = \frac{1}{2}\rho\mathbf{V}_s \quad (1.11)$$

where $\mathbf{V}_s = \frac{4}{3}\pi r^3$ is the volume of the sphere.

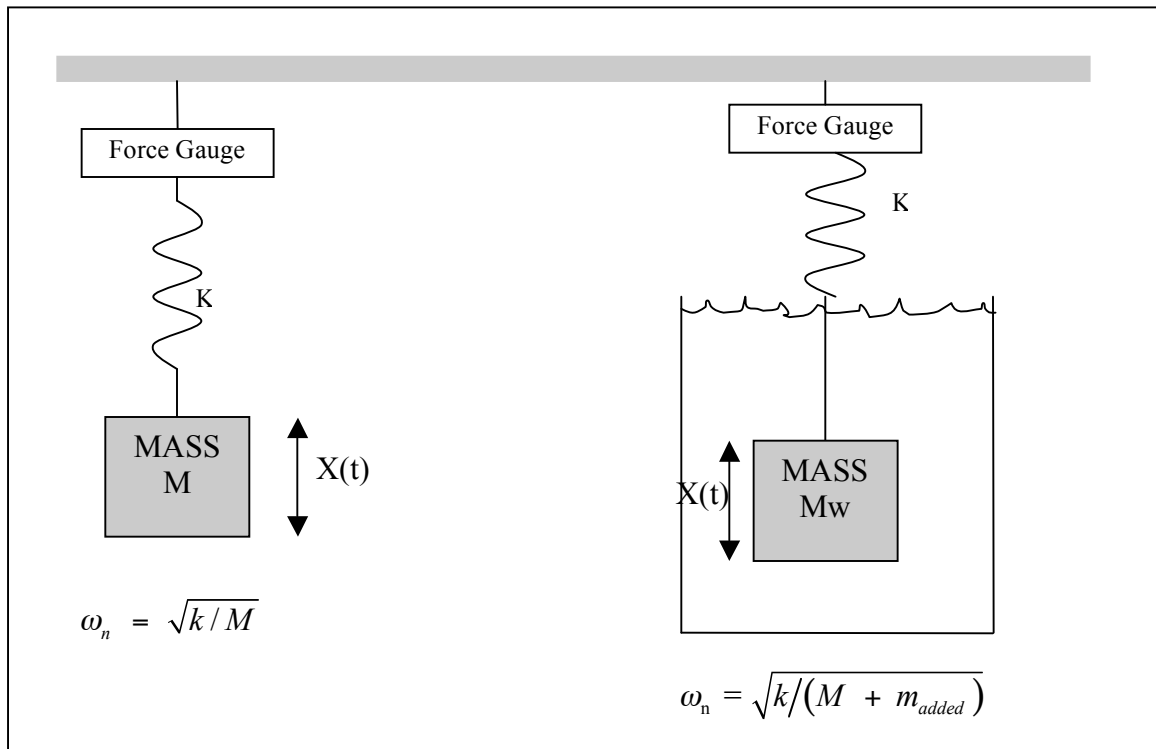
Cylinder:
$$m_a = \rho\pi a^2 L \quad (1.12)$$

where a is the cylinder radius and L is the cylinder length.

In this lab, you will be measuring the added mass of several objects. Using a simple, one degree-of-freedom spring-mass system with a strain gauge and an oscilloscope (that measures the voltage the strain gauge outputs), you will measure the natural frequency of oscillation in air and water. From this information you will be able to form an experimental measurement of the added mass of three different objects. You will compare your experimental results to the theory and comment on the differences that may arise.

Added Mass Experiment

Below is a picture of the idealized system set-up:



Note: We will be ignoring the viscous damping in this experiment. This approximation is acceptable for small amplitude oscillations in which inertial terms (accelerations) dominate.

Procedure:

1. Calibrate the force gauge

The force gauge measures load and outputs that as a voltage. In order to use the force gauge, you need to be able to relate the voltage to the load (via a calibration constant C [kg/V]). Hang 4 weights of known mass from the hook, and record the voltage output from the oscilloscope. Make a plot of voltage versus load.

2. Measure the dimensions of the three objects

Once you have finished the calibration, break into groups and choose an object to measure. (Each group measure one object, and share the measurements with the other groups for the post lab write-up.) Record the dimensions of the object you picked.

3. Measure weight and natural frequency of oscillation in air

To measure the mass, hang your object from the force gauge, and record the voltage output corresponding to its weight in air. To find the natural frequency, hang the object from the end of the spring and perturb it slightly. Observe the natural frequency data from the oscilloscope. Knowing the mass and natural frequency, you should now be able to calculate the spring constant, k . Keep in mind that k does not change when you oscillate to object in water.

4. Measure weight and natural frequency of oscillation in water

Use the cable extension to hang the object in the water. Repeat step 3.

5. Compute the added mass of the object

With the data you have gathered, you have everything you need to calculate an experimental value for the added mass of each object tested.

Lab Write Up

- Report the values for natural frequency of the systems (for all three objects) in both air and water. Physically explain the differences in the two frequencies.
- Compare the added mass value that you measured for each object and compare this to the theoretical value as determined by equation (1.11) or (1.12). Explain any differences between the values.
- For the experimental value of the added mass, determine the ratio between the experimental added mass and the mass of a volume of water equivalent to the size of your object. Does this make sense? Explain.
- How does buoyancy affect our system?