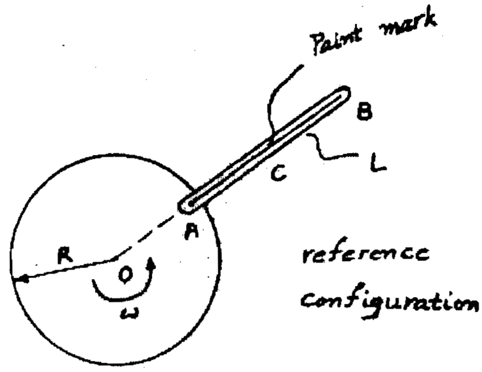
2.032 Dynamics

Problem Set No. 2

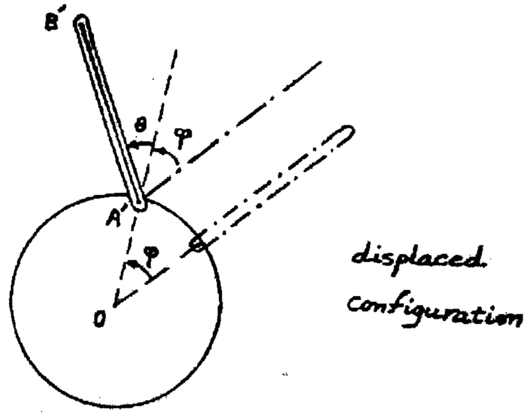
Problem 1

(a)

To find the angular velocity of the rod, compare orientation of AB to A′B′ :



$$\boldsymbol{\omega}_{rod} = (\dot{\varphi} + \dot{\theta})\,\hat{\mathbf{e}}_z = \underline{(\omega + \dot{\theta})\,\hat{\mathbf{e}}_z}$$

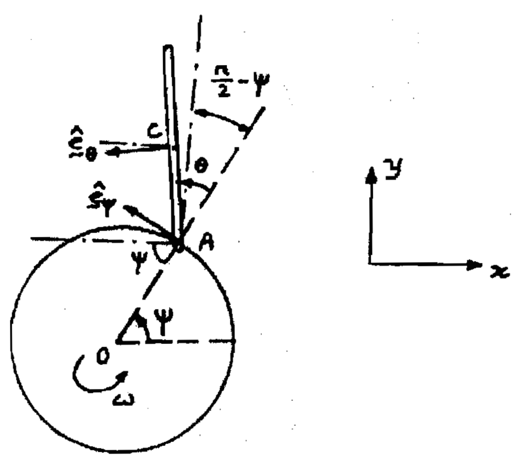


(b)

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{rod} \times \mathbf{AC}$$

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{OA} = \omega R\,\hat{\mathbf{e}}_\psi$$

$$\therefore \quad \mathbf{v}_C = \omega R\,\hat{\mathbf{e}}_\psi + (\omega + \dot{\theta})\frac{L}{2}\,\hat{\mathbf{e}}_\theta$$



$$\hat{\mathbf{e}}_\psi = -\sin\psi\,\hat{\mathbf{e}}_x + \cos\psi\,\hat{\mathbf{e}}_y$$

$$\hat{\mathbf{e}}_\theta = -\cos\left[\theta - \left(\frac{\pi}{2} - \psi\right)\right]\hat{\mathbf{e}}_x - \sin\left[\theta - \left(\frac{\pi}{2} - \psi\right)\right]\hat{\mathbf{e}}_y$$

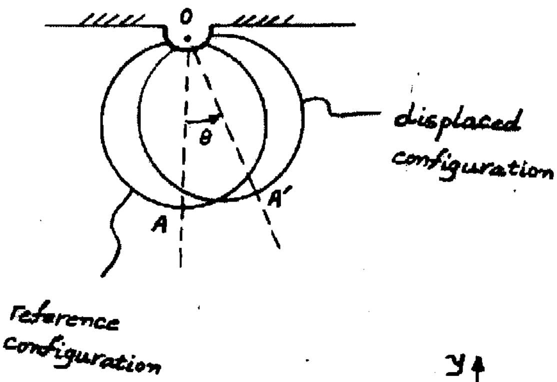
$$\hat{\mathbf{e}}_\theta = -\sin(\theta + \psi)\,\hat{\mathbf{e}}_x + \cos(\theta + \psi)\,\hat{\mathbf{e}}_y$$

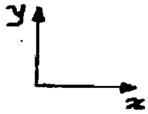
If $\psi(t=0) = \psi_0 \implies \underline{\psi(t) = \psi_0 + \omega t}$

So,

$$\underline{\mathbf{v}_C = \left[-\omega R \sin\psi - (\omega + \dot{\theta})\frac{L}{2}\sin(\theta + \psi)\right]\hat{\mathbf{e}}_x + \left[\omega R\cos\psi + (\omega + \dot{\theta})\frac{L}{2}\cos(\theta + \psi)\right]\hat{\mathbf{e}}_y}$$

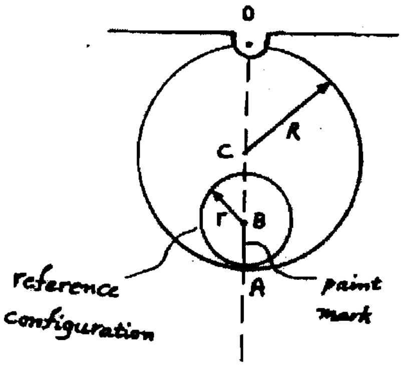
Problem 2

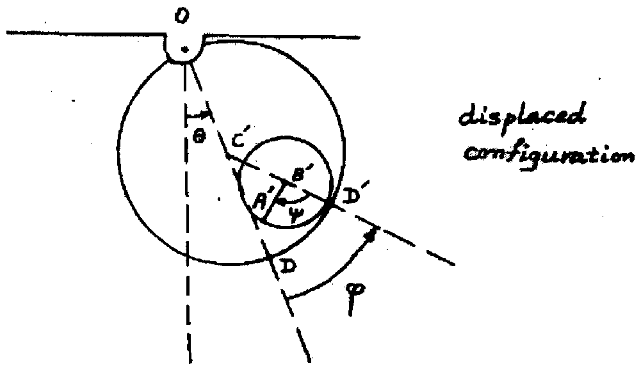


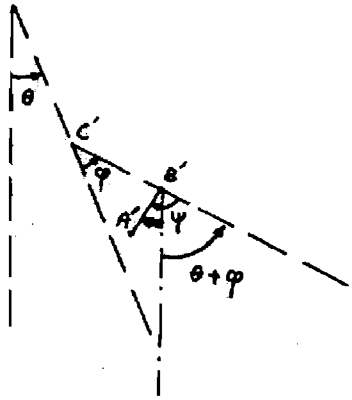


Comparing the orientation of OA to OA′, the angular velocity of the ring would be:

$$\underset{\sim}{\omega}_{ring} = \dot{\theta}\,\hat{\underset{\sim}{e}}_z$$







Comparing BA to B′A′,

$$\underset{\sim}{\omega}_{disk} = -\left[\dot{\psi} - (\dot{\theta}+\dot{\varphi})\right]\hat{\underset{\sim}{e}}_z$$

$$= (\dot{\theta}+\dot{\varphi}-\dot{\psi})\,\hat{\underset{\sim}{e}}_z$$

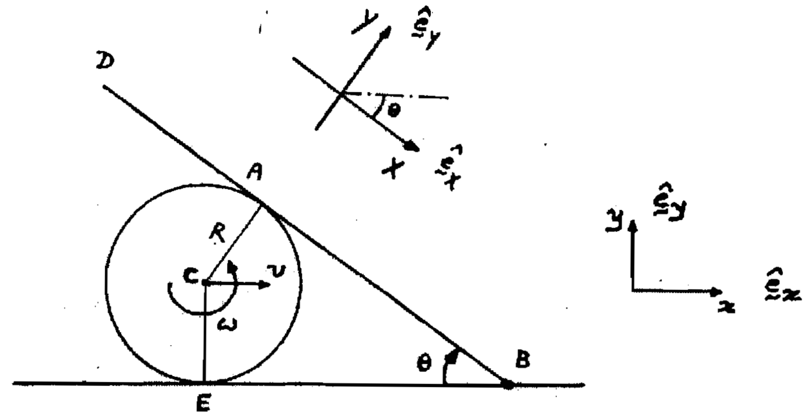
No slip $\Longrightarrow$ $\begin{cases} \underset{\sim}{v}_{D'}\big|_{disk} = \underset{\sim}{v}_{D'}\big|_{ring} \\ A'D' = DD' \end{cases}$ $\Longrightarrow$ $r\psi = R\varphi$ $\Longrightarrow$ $\psi = \dfrac{R}{r}\varphi$

$$\therefore \quad \underset{\sim}{\omega}_{disk} = \left[\dot{\theta} + \left(1-\frac{R}{r}\right)\dot{\varphi}\right]\hat{\underset{\sim}{e}}_z$$

Problem 3

(a)

$\underset{\sim}{v}_C = v\,\hat{\underset{\sim}{e}}_x = v\cos\theta\,\hat{\underset{\sim}{e}}_X + v\sin\theta\,\hat{\underset{\sim}{e}}_Y$



No slip between cylinder and the bar :

$$\underset{\sim}{v}_A\big|_{bar} = \underset{\sim}{v}_A\big|_{cylinder}$$

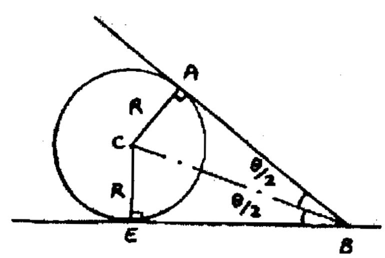
$$\underset{\sim}{v}_A\big|_{bar} = \underset{\sim}{\omega}_{bar} \times \underset{\sim}{BA} \quad\Longrightarrow\quad \underset{\sim}{v}_A\big|_{bar} = AB\,\dot\theta\,\hat{\underset{\sim}{e}}_Y$$

$$\underset{\sim}{v}_A\big|_{cylinder} = \underset{\sim}{v}_C + \underset{\sim}{\omega}_{cylinder} \times \underset{\sim}{CA}$$
$$= \left(v\cos\theta\,\hat{\underset{\sim}{e}}_X + v\sin\theta\,\hat{\underset{\sim}{e}}_Y\right) - R\,\omega_{cylinder}\,\hat{\underset{\sim}{e}}_X$$

$$\therefore \quad AB\,\dot\theta\,\hat{\underset{\sim}{e}}_Y = \left(v\cos\theta\,\hat{\underset{\sim}{e}}_X + v\sin\theta\,\hat{\underset{\sim}{e}}_Y\right) - R\,\omega_{cyl.}\,\hat{\underset{\sim}{e}}_X$$

$$\Longrightarrow \quad \left(v\cos\theta - R\,\omega_{cyl.}\right)\hat{\underset{\sim}{e}}_X + \left(v\sin\theta - AB\,\dot\theta\right)\hat{\underset{\sim}{e}}_Y = \underset{\sim}{0}$$

$$\Longrightarrow \quad \omega_{cylinder} = \frac{v\cos\theta}{R} \quad \& \quad \dot\theta = \frac{v\sin\theta}{AB}$$



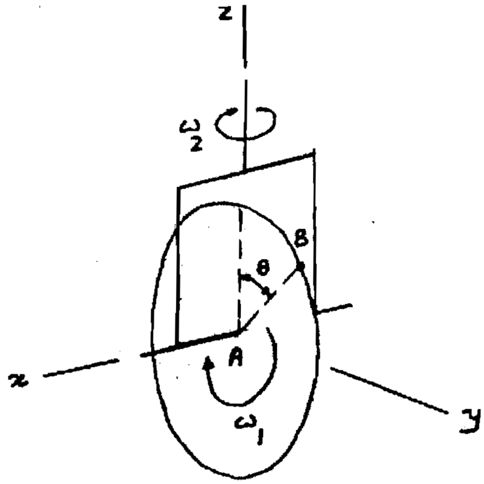
$$\frac{CA}{AB} = \tan\frac{\theta}{2} \quad\Longrightarrow\quad AB = R\cot\frac{\theta}{2}$$

So, $$\underset{\sim}{\omega}_{bar} = -\dot\theta\,\hat{\underset{\sim}{e}}_z = -\frac{v\sin\theta}{AB}\,\hat{\underset{\sim}{e}}_z = -\frac{v}{R}\sin\theta\tan\frac{\theta}{2}\,\hat{\underset{\sim}{e}}_z = \boxed{-\frac{2v}{R}\sin^2\frac{\theta}{2}\,\hat{\underset{\sim}{e}}_z}$$ Angular velocit of the bar BD

(b) $$\underset{\sim}{v}_E\big|_{cylinder} = \underset{\sim}{v}_C + \underset{\sim}{\omega}_{cylinder} \times \underset{\sim}{CE} = \left(v + \frac{v\cos\theta}{R}R\right)\hat{\underset{\sim}{e}}_x = \boxed{v\left(1+\cos\theta\right)\hat{\underset{\sim}{e}}_x}$$

velocity of the cylinder at the point where it contacts the ground.

Problem 4



Radius of the disk = R

$\underset{\sim}{v}_A = \underset{\sim}{0}$

$\underset{\sim}{v}_B = \underset{\sim}{v}_A + \underset{\sim}{\omega}_{disk} \times \underset{\sim}{AB} = \underset{\sim}{\omega}_{disk} \times \underset{\sim}{AB}$

$\underset{\sim}{AB} = R\left(\cos\theta\, \hat{\underset{\sim}{e}}_z + \sin\theta\, \hat{\underset{\sim}{e}}_y\right)$

$\underset{\sim}{\omega}_{disk} = -\omega_1 \hat{\underset{\sim}{e}}_x - \omega_2 \hat{\underset{\sim}{e}}_z$

$\underset{\sim}{v}_B = -\left(\omega_1 \hat{\underset{\sim}{e}}_x + \omega_2 \hat{\underset{\sim}{e}}_z\right) \times R\left(\cos\theta\, \hat{\underset{\sim}{e}}_z + \sin\theta\, \hat{\underset{\sim}{e}}_y\right)$

$\underset{\sim}{v}_B = R\omega_2 \sin\theta\, \hat{\underset{\sim}{e}}_x + R\omega_1 \cos\theta\, \hat{\underset{\sim}{e}}_y - R\omega_1 \sin\theta\, \hat{\underset{\sim}{e}}_z$ velocity of point B

Note that xy are rotating about z with ω_2.

$\underset{\sim}{a}_B = \dfrac{d\underset{\sim}{v}_B}{dt} = \dfrac{d\underset{\sim}{\omega}_{disk}}{dt} \times \underset{\sim}{AB} + \underset{\sim}{\omega}_{disk} \times \dfrac{d\underset{\sim}{AB}}{dt}$

$\dfrac{d\underset{\sim}{AB}}{dt} = \underset{\sim}{\omega}_{disk} \times \underset{\sim}{AB}$. $\dfrac{d\hat{\underset{\sim}{e}}_x}{dt} = -\omega_2 \hat{\underset{\sim}{e}}_y$ $\dfrac{d\hat{e}_y}{dt} = \omega_2 \hat{\underset{\sim}{e}}_x$

$\dfrac{d\underset{\sim}{\omega}_{disk}}{dt} = \omega_1\omega_2 \hat{\underset{\sim}{e}}_y$ $\dfrac{d\underset{\sim}{AB}}{dt} = R\omega_2 \sin\theta\, \hat{\underset{\sim}{e}}_x + R\omega_1 \cos\theta\, \hat{\underset{\sim}{e}}_y - R\omega_1 \sin\theta\, \hat{\underset{\sim}{e}}_z$

$\underset{\sim}{a}_B = \left(\omega_1\omega_2 \hat{\underset{\sim}{e}}_y\right) \times R\left(\cos\theta\, \hat{\underset{\sim}{e}}_z + \sin\theta\, \hat{\underset{\sim}{e}}_y\right) + \left(-\omega_1 \hat{\underset{\sim}{e}}_x - \omega_2 \hat{\underset{\sim}{e}}_z\right) \times R\left(\omega_2 \sin\theta\, \hat{\underset{\sim}{e}}_x + \omega_1 \cos\theta\, \hat{\underset{\sim}{e}}_y - \omega_1 \sin\theta\, \hat{\underset{\sim}{e}}_z\right)$

$\underset{\sim}{a}_B = 2R\omega_1\omega_2 \cos\theta\, \hat{\underset{\sim}{e}}_x - R\left(\omega_1^2 + \omega_2^2\right)\sin\theta\, \hat{\underset{\sim}{e}}_y - R\omega_1^2 \cos\theta\, \hat{\underset{\sim}{e}}_z$ acceleration of point B