

Problem Set No. 3

Out: Wednesday, September 23, 2009

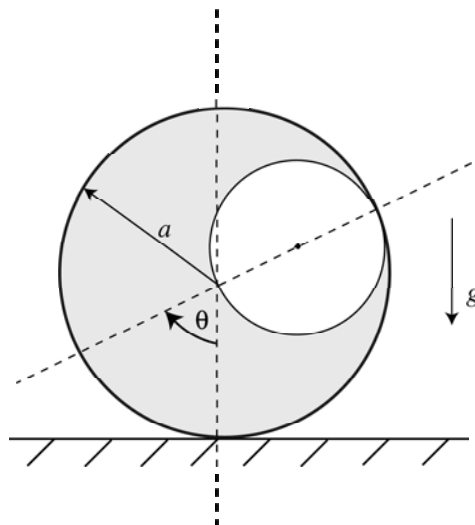
The homework problems are for practice only. Solutions are posted in a separate file. Please work on the problems and be prepared to ask questions related to this homework in the recitation of September 29, 2009 (4:00–5:30pm in Room 1-379).

Problem 1

A rigid circular cylinder of radius a has a hole of radius $\frac{1}{2}a$ cut out. Assume that the cylinder rolls without slipping on the floor.

(i) Compute the kinetic energy and the potential energy of the cylinder using the generalized coordinate θ defined below.

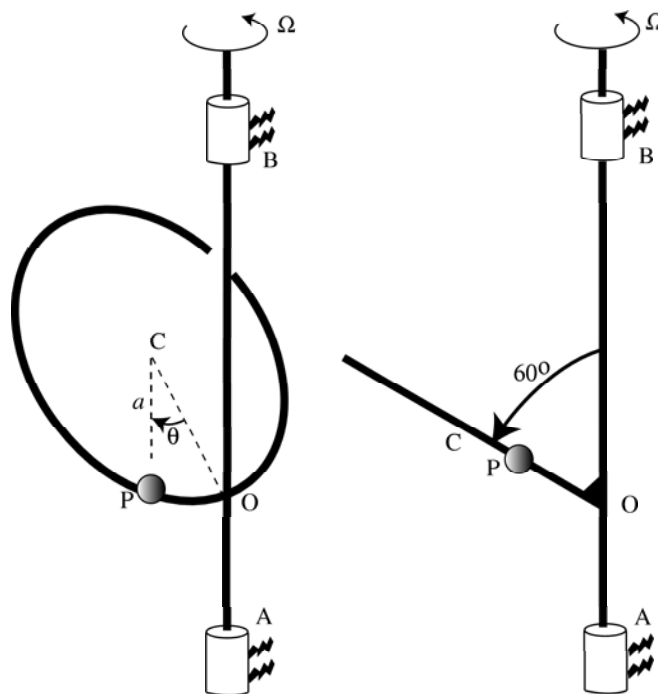
(ii) By suitably approximating the kinetic and potential energy expressions in (i), deduce the frequency of small rocking oscillations of the cylinder about the equilibrium position $\theta = 0$.



Problem 2 (adapted from PhD Qualifying Exam 2003)

A circular ring of radius a is fixed to a vertical shaft AB at O. The plane of the ring is inclined at 60° to the vertical as shown below so that the included angle COB is 60° , where C is the center of the ring. A particle P of mass m is free to slide without friction on the ring. The position of the particle is measured by the included angle OCP, denoted θ as shown. The shaft is driven at *constant* angular velocity Ω .

Find in terms of θ and $\dot{\theta}$ the velocity of P.



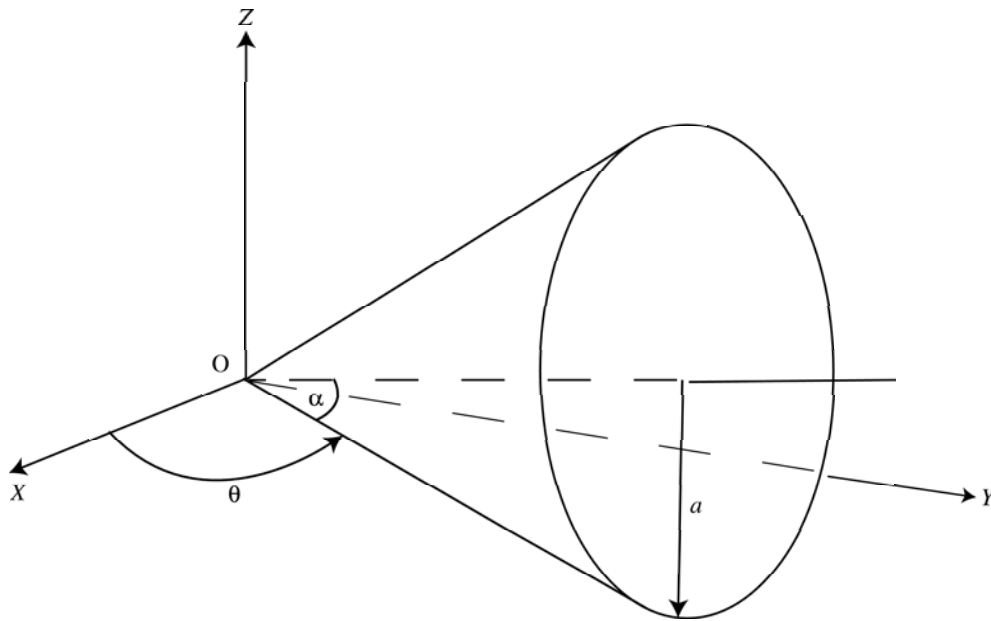
Problem 3

Determine the principal centroidal moments of inertia for the following homogeneous bodies:

- (a) a sphere of radius R ,
- (b) a circular cone of height h and base radius R .

Problem 4

Determine the angular velocity of a cone rolling on the XY -plane without slipping, as shown.



Problem 5

Find the kinetic energy of a homogeneous rigid cone rolling on a plane as in Problem 4 above.